No. of Printed Pages : 5

MINI-019

M.Tech. IN ADVANCED INFORMATION TECHNOLOGY – NETWORKING AND TELECOMMUNICATION (MTECHTC)

Term-End Examination

00013

June, 2015

MINI-019 : STATISTICAL SIGNAL ANALYSIS

Time : 3 hours *Maximum Marks : 100*

Note :

(i) Section I is compulsory.

*(ii) In Section II, solve any **five** questions.*

(iii) Assume suitable data wherever required.

(iv) Draw suitable sketches wherever required.

(v) Use of calculator is allowed.

SECTION I

1. Answer the following short answer questions : *10×3=30*

(i) What is if and only if condition for two sets A and B to be equal ?

(ii) When is an experiment called as a random experiment ? Give any two examples of random experiment.

(iii) Find the sample space S for the experiment of tossing a coin 4 times.

(iv) Consider an event in two parallel paths (P1 and P2) between two points A and B. P1 has two switches and P2 has one switch. Express the closed path event between the points A and B in terms of switches closed conditions.

(v) Two random variables X and Y are called independent if their joint cumulative distribution functions (*cdf's*) equals to what ?

(vi) State the central limit theorem. Brief its significance in probability theory.

(vii) What is the difference between Markov process and Markov chain ?

(viii) If the input to an LTI system is series of time shifted versions of impulse, what should be the output sequence ?

(ix) Describe the principle of operation of maximum likelihood estimator.

(x) Why does Shannon's sampling theorem lead to aliasing ? How can this be avoided ?

SECTION II

Attempt any ***five*** questions from this section :

2. Answer the following with mathematical equalities and examples : 7+7

   (i) Stochastic continuity, differentiation and integration of a random process.

   (ii) Autocorrelation, cross-correlation, white noise.

3. (a) State the elementary properties of probability. 6

   (b) A committee of 5 persons is to be selected randomly from a group of 5 men and 10 women. 4+4

      (i) Find the probability that the committee consists of 2 men and 3 women.

      (ii) Find the probability that the committee consists of all women.

4. (a) Derive a two state Markov discrete process. How is it used in digital communications ? 7

   (b) For the following transition matrix, find the steady-state probabilities : 7

$$\mathbf{T} = \begin{bmatrix} 0{\cdot}9 & 0{\cdot}1 \\ 0{\cdot}05 & 0{\cdot}85 \end{bmatrix}$$

5. Given a continuous random variable X with mean μ_x, variance σ_x^2 and pdf $f_x(x)$, where $f_x(x) = 0$ for $x < 0$. For any $a > 0$, *7+7*

(i) Show that $P(X \geq a) \leq \frac{\mu_x}{a}$ (Markov inequality)

(ii) Show that $P(|X - \mu_x| \geq a) \leq \frac{\sigma_x^2}{a^2}$ (Chebyshev inequality)

6. (a) Two manufacturing plants produce similar parts. Plant 1 produces 1,000 parts, 100 of which are defective. Plant 2 produces 2,000 parts, 150 of which are defective. A part is selected at random and found to be defective. What is the probability that it came from plant 1 ? *5*

(b) A lot of 100 semi-conductor chips contains 20 chips that are defective. Two chips are selected at random, without replacement, from the lot. *3×3=9*

(i) What is the probability that the first one selected is defective ?

(ii) What is the probability that the second one selected is defective given that the first one was defective ?

(iii) What is the probability that both are defective ?

7. The joint pdf of a bivariate random variables (X, Y) is given by

$$f_{x,y}(x,y) = \begin{cases} ke^{-(ax+by)} & x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

where, a and b are positive constants and k is a constant.

(i) Determine the value of k. *4*

(ii) Are X and Y independent ? *4*

(iii) Find the marginal pdf's $f_x(x)$ and $f_y(y)$. *6*

8. Consider a discrete-parameter random process $X(n) = \{X_n, n \geq 1\}$ where the X(n) are random variables with common cdf $f_x(x)$, mean μ, and variance σ^2.

(i) Find the joint cdf of X(n). *4*

(ii) Find the mean of X(n). *3*

(iii) Find the autocorrelation function $R_x(n, m)$ $R_d(n, m)$ of X(n). *4*

(iv) Find the auto covariance function $K_x(n, m)$ of X(n). *3*