No. of Printed Pages : 6 **MMTE-005**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. SC. (MACS)]

Term-End Examination
June, 2023

MMTE-005 : CODING THEORY

Time : 2 Hours *Maximum Marks : 50*

Note : *(i) Answer any* ***four*** *questions from question nos.* ***1*** *to* ***5****.*

(ii) Question ***No. 6*** *is compulsory.*

(iii) Use of calculator is ***not*** *allowed.*

(iv) Show all the relevant steps. Do the rough work at the bottom or at the side of the page only.

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1. (a) When do we say that the generator matrix of a $[n,k]$ linear code is in standard form ?

Check whether the generator matrix :

$$G=\begin{bmatrix}1 & 0 & 1 & 1 & 1\\ 0 & 1 & 0 & 1 & 1\\ 0 & 0 & 1 & 1 & 1\end{bmatrix}$$

of a linear code **C** is in standard form or not. Also, determine the length and dimension of **C** ? 3

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(b) Define permutation equivalence of linear codes. Check whether the codes $\mathbf{C}_1$ and $\mathbf{C}_2$ with generator matrices $G_1 = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ and $G_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$, respectively, are permutation equivalent. 2

(c) Let $n \in \mathbf{N}, q$ be a power of a prime and $0 \le s < n$. Define the q-cyclotomic coset of s modulo n. Find the 13-cyclotomic set of 1 modulo 17. 2

(d) Let $\mathbf{C}$ be a [7, 4] binary cyclic code with generator polynomial $x^3 + x + 1$. Find the generator matrix and the parity check matrix of the code. 3

2. (a) What is a repetition code ? If, in a repetition code in which a message of length two is sent thrice, the codeword 11 10 10 is received, decode the message assuming there is at most one error. 2

(b) Define a cyclic code, Check whether the code {110, 011, 101} is cyclic. 2

(c) Let $\mathbf{C}_1$ be the $\begin{bmatrix}4 & 3 & 2\end{bmatrix}$ binary linear code generated by :

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

and let $\mathbf{C}_2$ be the [4, 1, 4]-binary linear code generated by [1111]. Let **C** be the code obtained through using $(u \mid u + v)$ construction on the codes $\mathbf{C}_1$ and $\mathbf{C}_2$. Find the generator matrix of **C**. Also give the length and the dimension of **C**. 3

(d) Find the g.c.d. of $x^5 - x^4 + x + 1$ and $x^3 + x \text{ in } \mathbf{F}_5$. 3

3. (a) Construct the Tanner graph for the given parity check matrix H of an LDPC code. Further, does the graph contain a cycle ? Justify your answer : 5

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Let **C** be a self-dual [16, 8, 4] doubly even binary code. Find the weight enumerator of **C** by using Mac Williams identities. 5

4. (a) Show that the $\mathbf{Z}_4$-linear codes with generator matrices :

$$G_1 = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 2 \end{bmatrix} \text{ and } G_2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ 2 & 0 & 0 & 2 \end{bmatrix}$$

are monomially equivalent. 5

i	α^i	i	α^i
1	α	14	2α
2	α^2	15	$2\alpha^2$
3	$\alpha+2$	16	$2\alpha+1$
4	$\alpha^2+2\alpha$	17	$2\alpha^2+\alpha$
5	$2\alpha^2+\alpha+2$	18	$\alpha^2+2\alpha+1$
6	$\alpha^2+\alpha+1$	19	$2\alpha^2+2\alpha+2$
7	$\alpha^2+2\alpha+2$	20	$2\alpha^2+\alpha+1$
8	$2\alpha^2+2$	21	α^2+1
9	$\alpha+1$	22	$2\alpha+2$
10	$\alpha^2+\alpha$	23	$2\alpha^2+2\alpha$
11	$\alpha^2+\alpha+2$	24	$2\alpha^2+2\alpha+1$
12	α^2+2	25	$2\alpha^2+1$
13	2		

Table 1 : Powers of $\alpha \in \mathbf{F}_{27}$, where $\alpha^3+2\alpha+1=0$.

(b) If $x, y \in \mathbf{F}_2^n$, show that :

$$wt(x+y) = wt(x) + wt(y) - 2wt(x \cap y)$$

where $x \cap y$ is that vector in $\mathbf{F}_2^n$ which has 1 precisely those positions, where x and y have 1. Further, show that if **C** is a binary code with a generator matrix, each of whose rows have even weight, then every codeword of **C** has even weight. 5

5. (a) Let $\mathbf{F}_q$ have characteristic p. Prove that : 2

$$(\alpha + \beta)^p = \alpha^p + \beta^p$$

(b) Construct a [13, 10] BCH code over $\mathbf{F}_3$ with designed distance 2. Use $x^3 + 2x + 1 \in \mathbf{F}_3[x]$ as the primitive polynomial and Table 1. 5

(c) If a polynomial generator matrix of an [n, k] convolutional code **C** is basic and reduced, then prove that it is canonical. 3

6. Which of the following statements are true and which are false ? Justify your answer with short proof or a counter-example :

   (a) The code over $\mathbf{F}_3$ with generator matrix $\begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 & 1 \end{bmatrix}$ is self-orthogonal. 2

   (b) Every cyclic code is self dual. 2

   (c) $x^3 + x - 1$ is irreducible over $\mathbf{F}_5$. 2

   (d) There are two different codes with the same generator matrix. 2

   (e) The number of 3-cyclotomic cosets modulo 26 is 3. 2