No. of Printed Pages : 7 **MMT-007**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

[M. Sc. (MACS)]

Term-End Examination

June, 2023

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

Weightage : 50%

Note : *(i) Question No.* ***1*** *is compulsory.*

(ii) Attempt any ***four*** *questions out of Q. Nos.* ***2*** *to* ***7****.*

(iii) Use of scientific (non-programmable) calculator is allowed.

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1. State whether the following statements are True *or* False. Justify your answer with the help of a short proof or a counter-example. No marks are awarded for a question without justification :

2×5=10

(i) The function $f(x, y) = xy^2$ satisfies Lipschitz condition on :

$$a \leq x \leq b, -\infty < y < \infty$$

(ii) Fourier transform of :

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

is $\frac{\sin \alpha a}{\alpha}, \alpha \neq 0$.

(iii) For Legendre polynomial :

$$P'_n(x) = \begin{cases} \frac{1}{2} n(n+1) & ; \quad \text{for } x = 1 \\ (-1)^{n+1} \cdot \frac{1}{2} n(n+1); & \text{for } x = -1 \end{cases}$$

(iv) In applying Milne's or Adam-Bashforth method we require four starting values of y which are calculated by means of Picard's or Taylor's series method only.

(v) The order of the method :

$$u_{xx} = \frac{1}{h^2}[u(x+h, y) - 2u(x, y) + u(x-h, y)]$$

is three.

2. (a) Using method of Ferobenius, find the solution of the differential equation : 6

$$x^2 \frac{d^2y}{dx^2} + (x + x^2)\frac{dy}{dx} + (x - 9)y = 0$$

near $x = 0$.

(b) Find : 2

$$L^{-1}\left\{\frac{s}{(s^2+4)^2}\right\}$$

(c) Find $L\{F(t)\}$, if : 2

$$F(t) = \begin{cases} \sin\left(t - \frac{\pi}{4}\right), & t > \frac{\pi}{4} \\ 0, & t < \frac{\pi}{4} \end{cases}$$

3. (a) Find the solution of the initial boundary value problem : 5

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to given initial and boundary conditions :

$$u(x,0) = 2x \text{ for } x \in \left[0, \frac{1}{2}\right]$$

and 2 $(1 - x)$ for $\left[\frac{1}{2}, 1\right]$

$$u(0,t) = 0 = u(1,t).$$

You may use $h = 0.2$ and solve by Schmidt method with $\lambda = \frac{1}{6}$.

(b) Show that the method : 5

$$y_{i+1} = \frac{4}{3} y_i - \frac{1}{3} y_{i-1} + \frac{2h}{3} y_{i+1}$$

is A-stable when applied to test equation $y' = \lambda y, \lambda < 0$.

4. (a) Using Laplace transform, solve the equation : 5

$$\frac{\partial^2 u}{\partial t^2} = 9 \frac{\partial^2 u}{\partial x^2}$$

given that :

$$u(0,t) = u(2,t) = 0, \quad u_t(x,0) = 0$$

and $u(x,0) = 10 \sin 2\pi x - 20 \sin 5\pi x$.

(b) Using second order finite difference method, solve the boundary value problem :

$$\frac{d^2y}{dx^2} = \frac{3}{2}y^2 \text{ with } y(0) = 4, y(1) = 1$$

using $h = \frac{1}{3}$. 5

5. (a) Using Rodrigue's formula for Legendre polynomials, show that : 5

$$P_n(0) = \begin{cases} (-1)^{\frac{n}{2}} \dfrac{1.3.5.......(n-1)}{2.4.6.......n}, & \text{if n is even.} \\ 0 \quad , & \text{if } n \text{ is odd} \end{cases}$$

(b) Show that the equation : 3

$$x^2\frac{d^2y}{dx^2} + k^2xy = 0$$

reduces to a Bessel's equation in that substituting $y = \sqrt{x}\, z$ and then $t = \sqrt{x}$. Write its solution in Bessel's functions.

(c) Find $f(x)$, when : 2

$$\int_0^\infty f(x) \sin \alpha x \, dx = \begin{cases} 1-\alpha, & 0 \le \alpha \le 1 \\ 0 \quad , & \alpha > 1 \end{cases}$$

6. (a) Use finite Fourier transform to solve :

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 4, t > 0$$

Subject to the conditions :

$$u(x,0) = 2x, \quad 0 < x < 4$$

and $\quad u(0,t) = u(4,t) = 0$

for $0 < x < 4, t > 0$. 6

(b) Solve the boundary value problem :

$$y'' + y + f(x) = 0$$

$$y'(0) = 0, y(1) = 0$$

by determining the appropriate Green's function by using the method of variation of parameters and expressing the solution as a definite integral. 4

7. (a) Find the solution of the boundary value problem :

$$\nabla^2 u = x^2 + y^2, \ 0 \le x \le 1, \ 0 \le y \le 1$$

and $\quad u = \frac{1}{12}\left(x^4 + y^4\right)$

on the boundary using Galerkin method with rectangular elements and one internal node $\left(h = \frac{1}{2}\right)$. 6

(b) Solve the initial value problem :

$$y' = -2xy^2, \ y(0) = 1$$

with $h = 0.2$ on the interval $[0, .4]$. Use fourth order Runge-Kutta method. 4