

**M. Sc. (MATHEMATICS WITH
APPLICATIONS IN COMPUTER
SCIENCE)**

[M. Sc. (MACS)]

Term-End Examination

June, 2023

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours

Maximum Marks : 25

Note : (i) Question No. 1 is compulsory.

(ii) Attempt any three questions from
Question Nos. 2 to 5.

(iii) Use of calculator is not allowed.

1. State, giving reasons whether the following
statements are True or False : $5 \times 2 = 10$

P. T. O.

[2]

MMT-005

(a) If $f(z)$ is analytic on a domain D and $\operatorname{Re} f(z)$ is constant, then $f(z)$ is a constant.

(b) The equation $\cos z = 2$ has infinitely many solutions.

(c) The function $f(z) = e^{1/z^2}$ has a pole of order 2 at $z = 0$.

(d) The curve defined by $z(t) = 1 + \sin t$, $0 \leq t \leq 2\pi$, is a Jordan curve.

(e) If C is the contour $|z| = 2$ and

$$f(z) = \frac{\cos z}{(z+4)\left(2 - \frac{\pi}{2}\right)}, \text{ then } \oint_C f(z) dz = 0.$$

2. (a) Find the Mobius transformation mapping 0 to $-i$, i to -1 and ∞ to 1. 3

(b) Find the maximum modulus of $f(z) = e^y + 1$ on $|z| \leq 1$. 2

3. (a) Let $z(t) : [0, 1] \rightarrow \mathbb{C}$ be defined by : 2

$$z(t) = \begin{cases} 2it^2 - (8+i)t + 3, & \text{if } 0 \leq t \leq \frac{1}{2} \\ 3t - \frac{5}{2}, & \text{if } \frac{1}{2} \leq t \leq 1 \end{cases}$$

Find the integral of $f(z) = z$ along $z(t)$.

[3]

(b) Show that $u(x, y) = \sinh x \sin y$ is harmonic in entire complex plane \mathbb{C} . Also find the harmonic conjugate function of u . Is the harmonic conjugate function of u unique ?

3

4. (a) Find the Laurent expansion centered at 0 of $f(z) = \frac{1}{z^2 - z}$ in $|z| > 1$.

2

(b) Let C denote the circle $|z| = 2$ described in the counter-clockwise direction. Show that :

3

$$\left| \int_C \frac{\log 2}{z^2} dz \right| \leq x(\pi + \ln 2).$$

5. Show using residue theory that :

5

$$\int_{-\infty}^{+\infty} \frac{\cos ax}{1+x^2} dx = \pi e^{-a}, \quad a > 0.$$