BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA)

BASIC MATHEMATICS

Time : Three Hours

Maximum Marks : 100

Note : i) This question paper comprises of 3 Sections: Section A, B and C.

ii) Section A: Comprises of Short Answer Questions. Attempt any 5 out of 7 questions.

iii) Section B: Comprises of Medium Answer Questions. Attempt any 5 out of 7 questions.

iv) Section C: Comprises of Long Answer Questions. Attempt any 2 out of 3 questions.

Section-I (Short Answer Type Questions) (5×4=20)

Attempt any five questions. Each question carries 4 marks.

1. Show that $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & 0 \end{vmatrix} = 0$; where ω is a complex cube root of unity.

2. If $A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$, show that $A^2 - 4A + 5I_2 = O$.

3. Show that 133 divides $11^{n+2} + 12^{2n+1}$ for every natural number n.

4. If pth term of an Arithmetic Progression (A.P.) is q and qth term of the same A.P is p, find the rth term.

5. If α, β are roots of Quadratic equation $x^2 - 3ax + a^2 = 0$, find the value of a if $\alpha^2 + \beta^2 = \frac{7}{4}$.

6. If $y = \ln\left[\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right]$, find $\frac{dy}{dx}$.

7. Evaluate $\int x^2 \sqrt{5x-3}\, dx$.

Section-II (Medium Answer Type Questions) (5×10=50)

Attempt any five questions. Each question carries 10 marks.

8. a) If $1, \omega, \omega^2$ are cube roots of unity, show that :

$(2-\omega)(2-\omega^2)(2-\omega^{19})(2-\omega)^{23} = 49$.

b) If $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 3 \\ 3 & 0 & -1 \end{bmatrix}$, show that $A(\text{adj } A) = |A| I_3$.

9. a) Find the sum of all integers between 100 and 1000 that are divisible by 9.

b) Solve the equation $x^3 - 13x^2 + 15x + 189 = 0$, given that one of the roots exceeds the other by 2.

10. a) Solve the inequality $\frac{2}{|x-1|} > 5$ and graph the solution.

b) Find the points of local maxima and local minima of function $f(x) = x^3 - 6x^2 + 9x + 2014, x \in R$.

11. a) Using integration find length of the curve $y = 3 - x$ from (– 1, 4) to (3, 0).

b) Show that the lines $\frac{x-5}{4} = \frac{y-7}{-4} = \frac{z-3}{-5}$ and $\frac{x-8}{4} = \frac{y-4}{-4} = \frac{z-5}{4}$ intersect.

12. a) Use the principle of mathematical induction to prove that $\frac{1}{(1)(2)} + \frac{1}{(2)(3)} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$, for every natural number n.

b) Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$.

13. a) If $y = ax + \frac{b}{x}$, show that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$.

b) Find the sum of an infinite G. P. (Geometric Progression) whose first term is 28 and fourth term is $\frac{4}{49}$.

14. a) If $\vec{a}, \vec{b}, \vec{c}$ are coplanar, then prove that $\vec{a}+\vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are also coplanar.

b) Find vector and Cartesian equations of the line passing through the points (– 2, 0, 3) and (3, 5, – 2).

Section-III (Long Answer Type Questions) ($2\times15=30$)

Attempt any two questions. Each question carries 15 marks.

15. a) Find the points of discontinuity of the following function : $f(x) = \begin{cases} x^2, & x > 0 \\ x + 3, & x \le 0 \end{cases}$.

b) Evaluate the Integral (I) = $\int \frac{x^2}{(1+x)^3} dx$.

c) Draw the graph of the solution set for the following inequalities : $2x + y \ge 8; x + 2y \ge 8$ and $x + y \le 6$.

16. a) Write De Moivre's theorem, and use it to find $\left(\sqrt{5} + i\right)^3$.

b) A young child is flying a kite which is at a height of 50 m. The wind is carrying the kite horizontally away from the child at a speed of 6.5 m/s. How fast must the kite string be let out when the string length is 130 m ?

17. a) A product development company took the designing and development job of a product. The design job fetches the company Rs. 2,000 per hour and development job fetches the company Rs. 1,500 per hour. The company can devote at most 20 hours per day for designing and at most 15 hour per day for development of product. If total hours available for a day is at most 30, find the maximum revenue the product development company can get per day.

b) If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}; B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find a and b.