M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

June, 2022

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Question no.* ***1*** *is* ***compulsory****. Attempt any* ***three*** *questions from questions no. 2 to 5. Use of calculators is* ***not*** *allowed.*

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1. State, giving reasons, whether the following statements are *True* or *False* : *5×2=10*

(a) $f(z) = |z|$ is a nowhere differentiable function.

(b) If f is analytic on a domain D such that real part of f is constant in D, then the derivative of f is zero in D.

(c) If $f(z) = \dfrac{2z-1}{2-z}$, then $f(z)$ maps unit circle onto unit circle.

(d) $f(z) = \dfrac{\sin z}{\cos z}$ is not analytic in the domain $\{z : \dfrac{3\pi}{2} < \text{Re } z < \dfrac{5\pi}{2}\}$.

(e) $z = 0$ is a pole of order 2 for the function $f(z) = (1 + z + z^2)\, e^{-1/z}$.

2. (a) Let f(z) be an entire function such that there exist $M > 0$, $R > 0$ satisfying $|f(z)| \leq M|z|$ for $|z| > R$. Then show that f is a polynomial of degree one. *3*

(b) Determine analytic function whose real part is $\cos x \cosh y$. *2*

3. (a) Let $I(r) = \displaystyle\int_{\gamma} \frac{e^{iz}}{z}\, dz$, where $\gamma : [0, \pi] \to \mathbb{C}$ is defined by $\gamma(t) = re^{it}$, then $\displaystyle\lim_{r \to \infty} I(r) = 0$. *3*

(b) Let C be a closed contour on a domain D and $a \notin D$. Show that

$$\int_C \frac{1}{(z-a)^n}\, dz = 0 \text{ for } n \geq 2.$$ *2*

4. (a) Evaluate the integral $\int_C \frac{e^z - e^{-z}}{z^n} dz$, when n is a positive integer and $C(t) = e^{it}$, $0 \le t \le 2\pi$. *2*

(b) Find the bilinear transformation which takes the points 1, 0, ∞ to -1, i, $-$i. Also find the fixed points of the transformation, if any. *3*

5. Evaluate $\int_0^{2\pi} \frac{d\theta}{2 - \sin\theta}$. *5*