## MMT-009

## M. Sc. (MATHEMATICS WITH

 APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)
## Term-End Examination

June, 2021

## MMT-009 : MATHEMATICAL MODELLING

Time $: 1 \frac{1}{2}$ Hours Maximum Marxs : 25

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Note: (i) Attempt any five questions.
(ii) Use of scientific non-programmable calculator is allowed.
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1. (a) Let $\mathrm{P}(t)$, measured in kg , be the total mass or biomass of the fish population in a pond at time $t$. Write the continuous model for the population growth using logistic equation. The intrinsic growth rate $r$ and the carrying capacity $k$ are given the values 0.70 per year and $80.7 \times 10^{6} \mathrm{~kg}$ respectively. If the initial biomass is $\mathrm{P}_{0}=0.25 \mathrm{~K}$, find the biomass after 2 years.

Also, find the time $t$, for which $\mathrm{P}\left(t_{1}\right)=0.75 \mathrm{~K}$. 3
(b) Find the number of quantities required for estimating the expected return and standard deviation for 300 securities in Mar Bouritz model and how many estimates are required for these securities while using single index Sharpe model. 2
2. A locality is served by two malls. Each mall has two counters to serve the customers. Both the malls are equally popular and are known to have equal shares of the market. This is evident from the fact that customer's arrive at each mall's serving counter at the rate of 12 customers per hour. The average time to serve a customer is 05 minutes. Customer's arrival is according to a Poisson distribution and the service time is exponential. To provide better service to the customers, the owners of the two malls decide to consolidate into a single larger mall. What is the effect of consolidation on the waiting time of customers?
3. Consider the discrete population model governed by :

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$$
\mathrm{N}_{i+1}=\mathrm{N}_{i} \exp \left[r\left(1-\frac{\mathrm{N}_{i}}{k}\right)\right]
$$

where $i$ and $k$ are positive constants. Find out the steady states and perform the stability analysis corresponding to each steady state.
4. (a) "Indifference curves of an investor cannot intersect." Is this statement true ? Give reason for your answer.

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(b) Following is the data for number of years students studied a subject and score he/she received in that subject:

| Number of Years | Test Score |
| :---: | :---: |
| 3 | 57 |
| 4 | 78 |
| 4 | 72 |
| 2 | 58 |
| 5 | 89 |
| 3 | 63 |
| 4 | 73 |
| 5 | 84 |
| 3 | 75 |
| 2 | 48 |

Fit the least square line to this data. What is the score of the student who has studied for two years according to this line?
5. The transportation cost of 600 tons of a certain type of material from four factories $\mathrm{B}_{1}, \mathrm{~B}_{2}, \mathrm{~B}_{3}$ and $\mathrm{B}_{4}$ to three target stores $\mathrm{T}_{1}, \mathrm{~T}_{2}$ and $\mathrm{T}_{3}$ are given in the following table :

$$
\begin{aligned}
& \mathrm{T}_{1} \\
& \mathrm{~T}_{2}
\end{aligned} \mathrm{~T}_{3}
$$

The daily capacity of each of the factory is 150 per day and the daily requirements over each target store is 200 . Find the allocation for each factory to each target store which minimizes the total transport cost.

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6. (a) The return distribution on the two securities X and Y are given in the table below :

| Possible Rates of Return |  | Associated <br> Probability |
| :---: | :---: | :---: |
| X | Y | $\mathrm{P}_{x j}=\mathrm{P}_{y j}$ |
| 0.10 | 0.09 | 0.20 |
| 0.11 | 0.11 | 0.22 |
| 0.17 | 0.16 | 0.25 |
| 0.19 | 0.18 | 0.33 |

Find $\sigma_{\mathrm{XY}}$ and $\rho_{\mathrm{XY}}$.
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(b) The control parameter of growth and decay of a tumour are, respectively, 1200 and 600 per day. Also the damaged cells migrate due to vascularization of blood at the rate of 200 per day. Find the ratio of number of tumour cells after 10 days with the initial number of tumour cells. 2

