BACHELOR OF COMPUTER APPLICATIONS (BCA) (Revised)

Term-End Examination

June, 2021

BCS-012 : BASIC MATHEMATICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number* ***1*** *is* ***compulsory****. Attempt any* ***three*** *questions from the remaining questions.*

1. (a) If $A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$; $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, find a and b. *5*

(b) If the first term of an AP is 22, the common difference is – 4, and the sum to n terms is 64, find n. *5*

(c) Find the angle between the lines

$$\vec{r}_1 = 2\hat{i} + 3\hat{j} - 4\hat{k} + t(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\vec{r}_2 = 3\hat{i} - 5\hat{k} + s(3\hat{i} - 2\hat{j} + 6\hat{k}).$$ *5*

(d) If α, β are roots of $x^2 - 2kx + k^2 - 1 = 0$, and $\alpha^2 + \beta^2 = 10$, find k. *5*

(e) If $y = 1 + ln\,(x + \sqrt{x^2 + 1})$, prove that

$$(x^2 + 1)\,\frac{d^2y}{dx^2} + x\,\frac{dy}{dx} = 0.$$ 5

(f) Find the points of discontinuity of the following function : 5

$$f(x) = \begin{cases} x^2, & x > 0 \\ x + 3, & x \leq 0 \end{cases}$$

(g) Solve the inequality $\frac{5}{|x - 3|} < 7.$ 5

(h) Evaluate the integral

$$I = \int \frac{x^2}{(1 + x)^3}\,dx.$$ 5

2. (a) Use the principle of mathematical induction to show that

$2 + 2^2 + ... + 2^n = 2^{n+1} - 2$ for each natural number n. 5

(b) Using determinant, find the area of the triangle whose vertices are (1, 2); (– 2, 3) and (– 3, – 4). 5

(c) Draw the graph of the solution set for the following inequalities :

$$2x + y \geq 8,\ \ x + 2y \geq 8\ \text{ and }\ x + y \leq 6$$ 5

(d) Use De Moivre's theorem to find $(i + \sqrt{3})^3$. 5

3. (a) Find the absolute maximum and minimum of the following function : 5

$$f(x) = \frac{x^3}{x+2} \quad \text{on} \quad [-1, 1]$$

(b) Reduce the matrix $A = \begin{bmatrix} 5 & 3 & 8 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ to normal form and hence find its rank. 5

(c) If $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$; $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$; verify that

$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$. 5

(d) Find the length of function $y = 3 - 2x$ from $(0, 3)$ to $(2, -1)$ using integration. 5

4. (a) Find the quadratic equation with real coefficients and with the following pair of roots : 5

$$\left(\frac{m-n}{m+n}\right); \left(\frac{m+n}{m-n}\right)$$

(b) If $x = a + b$, $y = a\omega + b\omega^2$, $z = a\omega^2 + b\omega$ (where ω is a cube root of unity and $\omega \neq 1$), show that $xyz = a^3 + b^3$. 5

(c) Solve the following system of linear equations using Cramer's rule : 5

$$x + y = 0; \; y + z = 1; \; z + x = 3$$

(d) If $y = ln\left[e^x\left(\dfrac{x-2}{x+2}\right)^{3/4}\right]$, find $\dfrac{dy}{dx}$. *5*

5. (a) A software development company took the designing and development job of a website. The designing job fetches the company ₹ 2,000 per hour and development job fetches them ₹ 1,500 per hour. The company can devote at most 20 hours per day for designing and atmost 15 hours for development of website. If total hours available for a day is at most 30, find the maximum revenue the software company can get per day. *10*

(b) Evaluate $\int x\sqrt{3-2x}\ dx$. *5*

(c) Find the vector and Cartesian equations of the line passing through the points $(-2, 0, 3)$ and $(3, 5, -2)$. *5*