No. of Printed Pages : 7

MST-003

POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

June, 2020

MST-003 : PROBABILITY THEORY

Time : 3 Hours *Maximum Marks : 50*

Note : *(i) Question No. 1 is compulsory.*

*(ii) Attempt any **four** questions from the remaining (Questions Nos. 2 to 7).*

(iii) Use of scientific (non-programmable) calculator is allowed.

(iv) Use of formulae and statistical tables booklet for PGDAST is allowed.

(v) Symbols have their usual meanings.

P. T. O.

1. State whether the following statements are True *or* False. Give reasons in support of your answers : 2 each

(a) If odds against an event A are 2 : 5, then probability of occurrence of A is $\frac{3}{5}$.

(b) The function :

$$f(x, y) = \begin{cases} \frac{2}{5}x(2x + 3y), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{elsewhere} \end{cases}$$

is a joint probability density function.

(c) A random variable X follows Binomial distribution with mean 2 and variance 6.

(d) If X and Y are two independent random variables with probability density functions :

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}, -\infty < x < \infty$$

and $f(y) = \frac{1}{\sqrt{8\pi}} e^{-\frac{(y-5)^2}{8}}, -\infty < y < \infty$

respectively, then the variance of the random variable T = 2X + Y is 8.

(e) The probability of obtaining a total of 9 in a single throw with two dice is $\frac{1}{9}$.

2. (a) An integer is chosen at random from the first 200 positive integers. What is the probability that the interger chosen is divisible by 6 or 8 ? 5

(b) Three groups of children have respectively 3 girls and 1 boy, 2 girls and 2 boys and 1 girl and 3 boys. One child is selected at random from each group. What is the probability that three selected consist of 1 girl and 2 boys ? 5

3. (a) Verify that the following is probability density function : 3

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2, & -3 \le x \le -1 \\ \frac{1}{16}(6-2x)^2, & -1 \le x \le 1 \\ \frac{1}{16}(3-x)^2, & 1 \le x \le 3 \end{cases}$$

where the variable X is in the range (–3, 3).

(b) If a random variable X has the probability density function as follows :

$$f(x) = \begin{cases} \frac{1}{4} & -2 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

obtain the value of : 2, 3, 2

(i) $P(X < 1)$

(ii) $P(|X| > 1)$

(iii) $P[(2X + 3) > 5]$

4. (a) In a manufacturing process, a packaging machine produces 5% defective packages. Find the mean and the standard deviation of the number of defective packages in a random sample of 20 packages. 4

(b) Customers arrive at a photocopying machine at an average rate of two every 10 minutes. The number of arrivals is distributed according to a Poisson distribution. What is the probability that

there will be more than two arrivals during this time period ? 6

5. (a) Assume that the mean height of soldiers is 68.22 inches with $\sigma^2 = 10.8$ sq. inches. How many soldiers in a regiment of 1000 would you expect to be over 6 feet tall ? 5

(b) Subway trains on a certain line run every half hour between mid-night and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least 20 minutes ? 5

6. (a) Metro train arrives at a specified station at 10 minutes intervals starting at 5 A.M. that is they arrive at 5, 5 : 10, 5 : 20, 5 : 30 and so on. If a passenger arrives at the

station at a random time (that is uniformly distributed) between 5 A.M. and 5 : 20 A.M., find the probability that he waits : 6

(i) less than 5 minutes for the train.

(ii) at least 8 minutes for the train .

(b) If a boy is throwing stones at a target, what is the probability that his 10th throw in hit 5th hit, if the probability of hitting the target at any trial is $\frac{1}{2}$. 4

7. (a) In a certain assembly part, three machines B_1, B_2 and B_3 respectively make 30%, 45% and 25% of the products. It is known from the past experience that 2%, 3% and 2% of the products made by each machine, respectively, are defective. Now if we

suppose that a finished product is randomly selected, what is the probability that it is defective ? If the randomly selected product is found defective, what is the probability that it was made by machine B_1 ? 6

(b) A sample of 3 items is selected at random from a box containing 12 items of which 3 are defective. Find the expected number of defective items. 4