No. of Printed Pages : 4 **MMTE-001**

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M. Sc. (MACS)

Term-End Examination

June, 2020

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

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*Note : Question No. 7 is **compulsory**. Answer any **four** questions from Q. Nos. 1 to 6. Use of calculators is not allowed.*

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1. (a) Describe the Königsberg bridge problem. Model this using graphs. 4

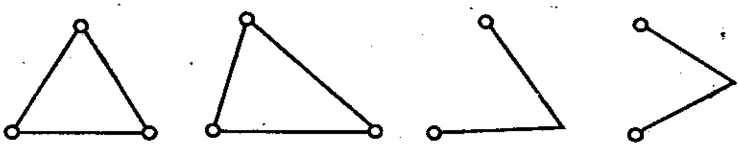
   (b) Draw a graph, with at least *four* vertices, of your own choice and write its adjacency matrix. 2

   (c) Give an example of a graph that is isomorphic to its complement. Also give an isomorphism from the graph to its complement. 4

2. (a) Prove that an edge in a graph is a cut-edge if it does not belong to a cycle. Use this to find the number of cut-edges in a tree of $m > 0$ edges. 4

(b) Prove that a k-regular ($k > 0$) bipartite graph has the same number of vertices in each partite set. 3

(c) Let G be a 4-vertex simple graph whose sub-graphs obtained by deleting one vertex are the following. Determine G : 3



3. (a) Prove that, in a non-trivial tree there is only one path joining any *two* of its vertices. 3

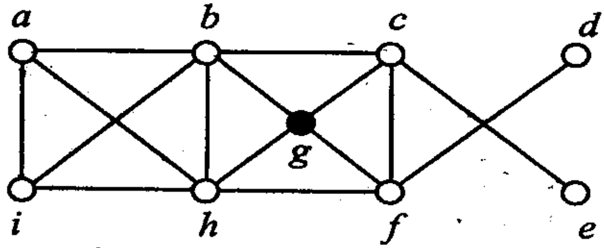
(b) If T is a tree of diameter 5, then prove that the diameter of T is at most 3. 3

(c) Check whether the sequence :

6, 6, 3, 3, 2, 2, 2, 2, 2, 2

is graphic or not. If it is, then draw a graph with this degree sequence. 4

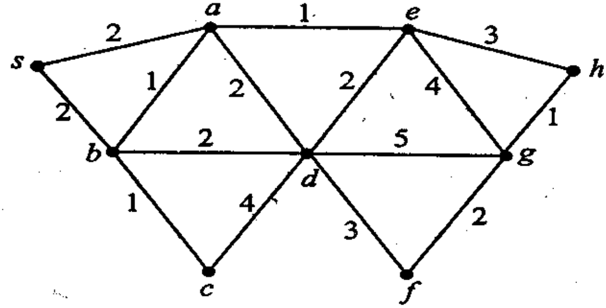
4. (a) Construct a maximum matching of the following graph :



Prove that the constructed matching is a maximum matching. 2

(b) If G is a bipartite graph, then prove that the maximum size of a matching in it is same as the minimum size of a vertex cover. 4

(c) Use Dijkstra's shortest path algorithm to find the shortest paths from s to all vertices of the following weighted graph G. Write down all the steps involved in finding the shortest paths. 4



5. (a) Describe Greedy colouring algorithm and prove that $\chi(G) \le \Delta(G) + 1$ for a graph G. 5

(b) Let G = (V, E) be a connected simple graph of order 10. Let T be a spanning tree of G having exactly 5 pendent vertices. Show that G has at least 5 vertices which are not cut-vertices. Deduce that every non-trivial simple connected graph has at least two vertices which are not cut-vertices. 5

6. (a) Draw the Peterson graph. Prove that it is non-planar. 5

(b) If G is a graph of order n at least 3, then prove that G is Hamiltonian if $\delta(G) \ge \frac{n}{2}$. 5

7. State whether the following statements are true *or* false by giving necessary justification :

$2 \times 5 = 10$

(a) Every self-complementary graph is connected.

(b) A graph with exactly one spanning tree is always a tree.

(c) Every graph has a perfect matching.

(d) For every graph G, $\chi(G) \le \Delta(G)$.

(e) Every Hamiltonian graph of order n has at least $n + 1$ spanning trees.