No. of Printed Pages : 4

**MMT-009**

# M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE

## Term-End Examination

## MMT-009 : MATHEMATICAL MODELLING

*Time : 1½ Hours]* *[Maximum Marks : 25*

*Weightage : 70%*

---

**Note:** 1. Do any five question.

2. Use oc calculators is not allowed.

---

1. (a) Formulate the model for which the reproductive function of the cancer cells in the tumour surface is given by : 10

$\Phi(c) = \frac{c-1}{1-2C}$, $c \neq \frac{1}{2}$ together with condition

$C = 20 \times 10^5$ at t=0.

Also find the density of the cancer cells in the tumour'surface at t = 45 days 3

(b) Given a set of seven cecurities with portfolio valves wi's $\frac{1}{3}, \frac{2}{3}, \frac{1}{2}, \frac{3}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}$. Find a suitable set of portfolio of these securities. 2

2. (a) Compare the risk of two securities 1 and 2 whose return distribution are given below:

| Possible rates of returns for security | | Associated probability |
|---|---|---|
| 1 | 2 | $P_{1j}$=P2j |
| 0.19 | 0.09 | 0.13 |
| 0.17 | 0.16 | 0.15 |
| 0.11 | 0.18 | 0.42 |
| 0.10 | 0.11 | 0.30 |

(b) In a population model of animals, the propationa birth rate and death rate are both constant, bare 0.20 per year and 0.50 per year verpectivley. Formative a model of population and discuss its long term behaviour. 2

3. Discuss the stability anlysis of the following model formulated to study the effect of toxicant on prey-predator population.

$$\frac{dN1}{dt} = ro\, N_1, -r, CoN_1 - bN_1N_2$$

$$\frac{dN2}{dt} = -do\, N_2, -d_1, V_0N_2 + \beta_0\, bN_1N_2$$

$$\frac{dCo}{dt} = -K_1\, P - g_1C_0, -m_1, V_0$$

$$\frac{dVo}{dt} = K_1\, P - g_2V_0, -m_1, V_0$$

$$\frac{dP}{dt} = Q - hP - KP(N_1 + B_2) + gC_0N_1 + eV_0N_2$$

Here $r_0$, $r_1$, b, $d_0$ $d_1$, $\beta_0$, $K_1$, $K_2$, $g_1$, $g_2$, $m_1$, $m_2$, Q,h, K,g, l and all positive controls.

$N_1(t)$= density of prey population

$N_2(t)$= density of predoter population

$C_0(t)$= concentration of the toxicant in the individuals of the prey population.

V(t)= concentration of the toxicant in the individuals of the predator populatione

P(t)= concentration of the toxicant in the environment

Q=costant input rate,

h= decay rate,

K=Ingertion rate of toxicant by the population

g.l= return rate of toxcant in the environment after the death of the population, assuming toxicant is non-degredable

$r_0$, $r_1$ are birth rates, do the death rate, both predation rate,

$B_0$ is anversion coefficient, $m_1$, $m_2$ and deperation rates, $K_1$, $K_2$ are uptek rate and $g_1$, $g_2$ are loss rates.

4. Consider the population model given by the difference equation

$U_{n+1}$= r$U_n$ (2-$U_n^2$, r>0) Find the steady states of the population and discuss the linear stability for

(i) $0 < r > ½$

(ii) $½ < r < 1$

What would you expect to happen when $r > 2$ ?

5. A television company produces two models $T_1$, and $T_2$ which have profit contributions 2 (Rs in thousands) and 3 (Rs in thousands) per unit production. respectively. Each type of television requires a certain amount of time for the manufacture of components and assembling. One unit of model $T_1$ requires 6 hours for manufacturing and 1 hour for assembling. The corresponding figures for one unit of model $T_2$ are 5 and 3, respectively. The company will able to make available 25 hours for manufacturing and 10 hours for assembling. Obtain the optional production schedule for the Company using Branch and Bound method. 5

6. Explain each of the following with examples : 5

   (i) Harwitz criteria

   (ii) Multiple linear repression model with predictors.

   (iii) Reaction diffusion model verses Advection reaction diffusion model.

   (iv) Variational matrix or Jacobian of a system of n differential equations.

   (v) Data Visualisation.

—x—