No. of Printed Pages : 8

MMT-008

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE

Term-End Examination

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 Hours] *[Maximum Marks : 100*

Note: 1. Q. No. 8 is compulsory.

2. Attempt any six questions from Q No. 1 to 7.

3. Use scientific non prgramable calculators is allowed.

4. Symbols have their usual meanings.

1. (a) Consider the Markov chain having the following transition probability matrix : 10

$$P = \begin{bmatrix} 1/3 & 0 & 2/3 & 0 & 0 & 0 \\ 0 & 1/2 & 1/4 & 0 & 1/4 & 0 \\ 2/5 & 0 & 3/5 & 0 & 0 & 0 \\ 0 & 1/4 & 1/4 & 1/4 & 0 & 1/4 \\ 0 & 0 & 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 0 & 0 & 1/4 & 3/4 \end{bmatrix}$$

(i) Draw the digram of the Markov chain.

(ii) Write the classes of persistent, non-null and aperiodic states.

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1. (a) Consider the Markov chain having the following transition probability matrix : 10

$$P = \begin{bmatrix} \frac{1}{3} & 0 & \frac{2}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} & 0 \\ \frac{2}{5} & 0 & \frac{3}{5} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

(i) Draw the digram of the Markov chain.

(ii) Write the classes of persistent, non-null and aperiodic states.

(iii) Find the probability of absorption to the classes. Also, find the mean time upto absorption from transient state 2 to 4.

(b) Find the principal components and proportions of total population variance explained by each component when the covariance matrix is.

$$\Sigma = \begin{bmatrix} 5 & 2 \\ 2 & 7 \end{bmatrix}$$ 5

2. (a) A communication system transmits two digits 0 and 1, each of them passing through several stages. Suppose the probability at the time of leaving of the digit that enters remain unchanged is a p and when it changes is 1-p. Suppose that X_0 is the digit that enters in the first stage of the system and X_n $(n \leq 1)$ is the digit O leaving in the nth stage of the system. Show that $\{x_n, n \geq 1\}$ forms a Markov chain. Find P, P², P³ and compute $P\left[X_2 = 0 | Xo = 1\right]$ and $P\left[X_3 = 1 | Xo = 2\right]$.

Let $\underset{\sim}{X}_n N_4(\underset{\sim}{u}, \underset{\sim}{\Sigma})$ with

$$\underset{\sim}{u} = \begin{bmatrix} 2 \\ 4 \\ -1 \\ 3 \end{bmatrix} \quad \text{and} \quad \underset{\sim}{\Sigma} = \begin{bmatrix} 4 & 2 & -3 & 4 \\ 2 & 4 & 0 & 1 \\ -3 & 0 & 4 & -2 \\ 4 & 1 & -2 & 8 \end{bmatrix}.$$

Suppose $\underset{\sim}{Y}$ and $\underset{\sim}{Z}$ are two partitioned sub-vectors of $\underset{\sim}{X}$ such that $\underset{\sim}{Y}' = [x_1 x_2]$ and $\underset{\sim}{Z}' = [x_3 x_4]$ find. 7

(i) E (y/z)

(ii) Cov. (y/z)

(iii) Correlation coefficient $r_{12.34}$. 8

3. (a) The number of accidents in a town follows a poisson process with a mean of 2 per day and the number X_i of people involved in the ith accident be the iid given as

$P[xi = b] = \frac{1}{2^k}; k \geq 1.$ Find the mean and variance of the number of people involved in accident per week. 5

(b) Suppose 10 and 15 observations are made on two random variables $\underset{\sim}{X}_1 \sim N_2(\underset{\sim}{u}_1, \underset{\sim}{\Sigma})$ and $\underset{\sim}{X}_2 \sim N_2(\underset{\sim}{u}_2, \underset{\sim}{\Sigma})$, where $\underset{\sim}{lu} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \underset{\sim}{u}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and

$\underset{\sim}{\Sigma} = \begin{bmatrix} 1.5 & 2 \\ 2 & 4 \end{bmatrix}$. Considering equal cost and equal prior probabilities, classify the observation $[1.5, 1]$ in one of the two populations. 10

4. (a) Find the differential equation of pure death process. If the process Narts with i individuals, find the mean and variance of the number N(t) present at time t. 6

(b) Define conjoint analysis with suitable example. How conjoint analysis is used to optimize product design? 5

(c) The inter occurence time in a renewal process follower exponential distribution with rate $\lambda > 0$. What will be the distribution of number of renewals in time t? Obtain renewal fucntion and its a place transform. 4

5. (a) Patients arrive at the outpatient department of a hospital in accordance with a poisson process at the mean rate of 12 per hour, and the distribution of time for examination by an attending physician is exponential with mean of 10 minutes. What is the minimum number of physicians to be posted for ensuring a steady state distribution. For this number, find.

(i) The expected waiting time of a patient prior to being examined.

(ii) The expected number of patients in the out-patient department. Also, find the average number of physicians who remain idle.

(b) Find all stationary distributions for a Markov chain having the following tarnsition probability matrix : $P = \begin{bmatrix} 0.4 & 0.6 \\ 0.7 & 0.3 \end{bmatrix}$ 6

6. (a) Suppose that each individuatl at the end of the unit of time produces either k (k >, 2) or O direct descendants with probabilities p or q, respectively. Check whether the probability of ultimate extinction is less than 1. Find this probability for K=3 when þ=q=½. 6

(b) Let $\underset{\sim}{X} \sim N_3(u, \Sigma)$ with the date matrix

$$X = \begin{bmatrix} 5 & 3 & 4 \\ 2 & 1 & 3 \\ 5 & 6 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$ Find the variance-covariance matrix $\underset{\sim}{\Sigma}$ and the correlation matrix $\underset{\sim}{R}$. 9

7. (a) The joint probability mass function of random variables X and Y is given by :

y \ x	0	1	2
-2	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$
0	$\frac{1}{6}$	0	$\frac{1}{12}$

(i) Find the marginal distributions of X and Y.

(ii) Find E (X) and E (Y).

(iii) Find Cove (X,V).

(b) Consider M/M/1 queveing system with arriva rate λ and service rate 2 λ and another M M/2 queveing system with arrival rate λ a service rat λ. Show that the average wait time in system M/M/1 is smaller than waiting time in M/M/2 system.

(c) Define canonical correlation with suitai example. How canonical correlation is us to do optional scaling.

8. State which of the following statements are tr and which are false. Give a short proof or counter example in support of your answer. 1

(i) Two independent events B and C are such that $P(B \cap C) > 0$, then $P(A/B \cap C) = P(A/B).P(A/C)$.

(ii) If X and Y are two random variables with V(X)=V(Y)=2, then –2<Cov (X,Y) <2.

(iii) The row sums in the infinitesimal generator of a birth and death process are zero.

(iv) A real symmetric matrix $(a_{ij})_{n \times n}$ with $a_{ii}=-1$ cannot be positive definite.

(v) The maximum likelihood estimator of $u'\Sigma^{-1}u$ is $\bar{X}'U^{-1}\bar{X}$, where $\bar{X}$ and U are the maximum likelihood estimators of u and Σ respectively.

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