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MMT-007

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M. Sc. (MACS)

Term-End Examination

June, 2020

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours | *Maximum Marks : 50*

Note : *(i) Question No. 1 is compulsory.*

(ii) Answer any four questions out of Q. Nos. 2 to 7.

(iii) Use of scientific non-programmable calculator is allowed.

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1. State whether the following statements are true *or* false. Justify your answer with the help of a short proof or a counter example. No marks are awarded for a question without justification :

2 each

(a) The Lipschitz constant for the function $f(x, y) = x^2 |y|$, defined on $|x| \le 1, |y| \le 1$ is equal to 1.

(b) The interval of absolute stability of the 2nd order classical Runge-Kutta method to solve the initial value problem :

$$y' = \lambda y$$
$$y(x_0) = y_0$$

where $\lambda < 0$ is $]-2, 0[$.

(c) If H_n is a Hermit polynomial of degree n, then :

$$H_{2n}(0) = \frac{(-1)^n \underline{|2n+1}}{\underline{|n+1}}$$

(d) $L[t^2 \sin 5t] = \dfrac{3s^2 - 25}{(s^2 + 25)^3}$, where L is Laplace transform.

(e) The order of the method :

$$u_{xx} = \frac{1}{h^2}[u(x+h, y) - 2u(x, y) + u(x-h, y)]$$

is three.

2. (a) Find a series solution about $x = 0$ of the differential equation : 6

$$4(x^4 - x^2)y'' + 8x^3 y' - y = 0$$

(b) Solve the initial value problem : 4

$$y' = -2xy^2$$

$$y(0) = 1$$

with $h = 0.2$ on the interval [0, 0.4]. Use fourth order Runge-Kutta method.

3. (a) Solve : 5

$$\frac{\partial u}{\partial t} = 2\frac{\partial^2 u}{\partial x^2}$$

given that :

$$u(0, t) = 0$$

$$u(5, t) = 0$$

$$u(x, 0) = \sin(\pi x)$$

using Laplace transform.

(b) Find the truncation error and the order of the method : 5

$$u_{i+1,j} + u_{i-1,j} + u_{i,j+1} + u_{i,j-1} - 4u_{i,j} = h^2 G_{ij}$$

for the Poisson equation :

$$u_{xx} + u_{yy} = G(x, y).$$

4. (a) Construct Green's function for the b.v.p. : 5

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 \; 0 < x < \frac{\pi}{2},$$

$$y(0) = 0, \; y\left(\frac{\pi}{2}\right) = 0.$$

(b) Using Inverse Fourier transform, find $f(x)$ if : 3

$$F_c(\alpha) = \begin{cases} \left(a - \frac{\alpha}{2}\right) & \alpha \le 2a \\ 0 & \alpha > 2a \end{cases}$$

(c) Find the Laplace inverse of $\cot^{-1} s$. 2

5. (a) Find the solution of the boundary value problem : 5

$$\nabla^2 u = x^2 + y^2$$

$$0 \le x \le 1$$

$$0 \le y \le 1$$

subject to the boundary conditions :

$$u = \frac{1}{12}(x^4 + y^4)$$

on the lines $x = 1, y = 0, y = 1$ and :

$$12u + \frac{\partial u}{\partial x} = x^4 + y^4 + \frac{x^3}{3}$$

on $x = 0$ using the five point formula. Assume $h = \frac{1}{2}$ along both axes. Use central difference approximation in the boundary condition.

(b) Find : 5

$$L(\sin \sqrt{t})$$

where L denotes Laplace transform. Deduce the value of $L\left(\frac{\cos \sqrt{t}}{\sqrt{t}}\right)$.

6. (a) Find the solution of the initial boundary value problem : 5

$$u_t = u_{xx}$$

$$0 \leq x \leq 1$$

$$t > 0$$

with conditions :

$$u(x,0) = \begin{cases} 2x, & 0 \le x \le \frac{1}{2} \\ 2(1-x), & \frac{1}{2} \le x \le 1 \end{cases}$$

$u(0, t) = 0 = u(1, t)$, $t > 0$ using Crank-Nicholson method with $\lambda = \frac{1}{2}$. Assume $h = \frac{1}{4}$ and interpret for one time level.

(b) Find $y(0.1)$, $y(0.2)$ and $y(0.3)$ for the equation : 5

$$\frac{dy}{dx} = x^2 - y,$$

$y(0) = 1$ by using fourth order Taylor series method. Hence obtain $y(0.4)$ using Adam-Bashforth method with :

$$P : y^p_{n+1} = y_n + \frac{h}{24}(55y'_n - 59y'_{n-1} + 37y'_{n-1} - 9y'_{n-3})$$

$$C : y^c_{n+1} = y_n + \frac{h}{24}(9y'_{n+1} - 19y'_n - 5y'_{n-1} + y'_{n-2}).$$

7. (a) Using the substitution $z = \sqrt{x}$, reduce the equation : 3

$$xy'' + y' + \frac{y}{4} = 0$$

to Bessel equation and hence write its solution.

(b) Using the generating function for Legendre polynomials P_n, $n = 0, 1, 2, \ldots$ prove that : 4

$$1 + \frac{1}{2}P_1(\cos\theta) + \frac{1}{3}P_2(\cos\theta) + \ldots\ldots = \ln\left[\frac{1+\sin\left(\frac{\theta}{2}\right)}{\sin\left(\frac{\theta}{2}\right)}\right]$$

(c) Using Convolution theorem, find the Fourier inverse of the function : 3

$$\frac{1}{(i\alpha + k)^2}, k > 0$$

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