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MMT-005

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M. Sc. (MACS)

Term-End Examination

June, 2020

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : *(i) Question No. 1 is **compulsory**.*

*(ii) Attempt any **three** Question Nos. 2 to 5.*

*(iii) Use of calculator is **not** allowed.*

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1. State, giving reasons whether the following statements are True *or* False : 2 each

(a) $|\cos z| \leq 1$, for all $z \in \mathbf{C}$

(b) The function :

$$f(z) = z^2 + \frac{1}{z^2}$$

is conformal at $z = \dfrac{\pm 1 + i}{\sqrt{2}}$.

P. T. O.

(c) $z = 2$ is the fixed point of the Mobius transformation :

$$T(z) = \frac{3z - 4}{z - 1}.$$

(d) $\int_C f(z)\, dz \neq 0$ for every closed curve C inside $1 \leq |z| \leq 2$ where $f(z) = \frac{1}{z}$.

(e) $\frac{1}{\sin \frac{1}{z}}$ is mesomorphic.

2. (a) Find the Laurent series expansion of the function : 3

$$f(z) = \frac{z}{(z+1)(z+2)}$$

in the annular region :

$$\frac{1}{2} < |z+1| < \frac{3}{4}.$$

(b) Find zeros and poles of the function : 2

$$\frac{\sin z}{z - z^2}$$

3. (a) Find the harmonic conjugate $v(x, y)$ of the function : 2

$$u(x, y) = x^2 - y^2$$

if $f(z) = u(z) + iv(z)$ is an analytic function. Find the function f also.

(b) Find the Mobius transformation such that : 3

$$T(-i) = -i$$

$$T(1) = 0$$

$$T(i) = i$$

4. (a) Verify the maximum and minimum modulus theorem for e^z on $|z| \leq 1$. 3

(b) Find all the solutions of : 2

$$e^z = 1 + i$$

5. Evaluate : 5

$$\int_0^{\infty} \frac{\cos x}{1 + x^2}\, dx$$

using contour integration.

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