No. of Printed Pages : 4 **MMT-003**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M. Sc. (MACS)

Term-End Examination

June, 2020

MMT-003 : ALGEBRA

Time : 2 Hours *Maximum Marks : 50*

Note : *Question No. 1 is **compulsory**. Answer any **four** questions from Q. No. 2 to 6. Use of calculator is **not** allowed.*

1. State whether the following statements are True *or* False. Give reasons for your answers : 10

   (i) There exists a field of order 26.

   (ii) Any *two* elements of order 3 in S_7 are conjugates.

   (iii) Every group of order 15 is abelian.

   (iv) Every free abelian group is a free group.

   (v) Z $[x]$ has finitely many units.

2. (a) If :

$$G = (\{a\}, \{g_0\}, \{g_0 \to a^3, g_0 \to a^5 g_0\}, g_0)$$

find L (G). 3

(b) Find the Legendre symbol $\left(\frac{18}{41}\right)$. 2

(c) Check whether or not : 5

$$Q\left(2^{1/3}\right) = Q\left(4^{1/3}\right)$$

Also obtain $[Q(2^{1/3}) : Q]$.

3. (a) Let G be a group of order 51. Suppose that G acts on a set X having 19 elements. What are the possible values of $|O_x|$ for $x \in X$? (O_x is the orbit of x under this action). Show that there exists an x_0 in X such that $O_{x_0} = \{x_0\}$. 5

(b) Find the elementary divisors and invariant factors of the group $Z_6 \times Z_{14} \times Z_{15}$. Also find the highest order an element of this group can have. 5

4. (a) Find a Sylow 5-subgroup of S_5. How many such subgroups are there ? How many 5-cycles are there in S_5 ? Give reasons for your answer. 6

(b) Check whether or not 978-93-80250-72-5 is a valid ISBN number. 2

(c) Let : 2

$$R = \frac{Z_7[x]}{\langle x^2 + T \rangle}$$

Check whether or not this is the splitting field of a polynomial over Z_7.

5. (a) Check whether or not Q $(5^{1/4})$ is a normal extension of Q. Is it a normal extension or Q $(\sqrt{5})$? Give reasons for your answer. 4

(b) Find the order of the group : 3

$$Z(A) = \{X \in GL_2(Z_7) \mid AX = XA\}$$

where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

(c) Check whether or not : 3

$$\rho(m) = \begin{pmatrix} 1 & m \\ 0 & 1 \end{pmatrix}$$

is a representation or Z. Further, give an example of a 1-dimensional representation of Z, with justification.

6. (a) Find the stabilizer of :

$$\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \leftarrow M_2(R)$$

under left multiplication action of $GL_2(R)$ on $M_2(R)$. 2

(b) Let G be the group generated by x, y, z with the only relation $xyx^{-1}yz^{-1}$. Show that G is a free group. 2

(c) Show that :

$$f(x) = x^2 + x + 2 \in Z_3[x]$$

is irreducible. Further, find the order of $f(x)$. 4

(d) Check whether or not [C : R] is an algebraic extension. 2

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