No. of Printed Pages : 4 **MMT-002**

M. Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) M. Sc. (MACS)

Term-End Examination

June, 2020

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ Hours *Maximum Marks : 25*

Note : *Question No. 5 is* ***compulsory****. Answer any* ***three*** *questions from 1 to 4. Calculators are* ***not*** *allowed.*

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1. Find the Jordan canonical form of : 5

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Also find a matrix P so that $P^{-1}AP$ in Jordan form.

2. (a) Find the best least squares quadratic polynomial fit to the data below : 3

x	y
–1	0
0	1
1	3
2	9

(b) Evaluate exp (A^2) if : 2

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

3. (a) Determine if the following matrix is positive definite or not : $1\frac{1}{2}$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(b) Let M and D be a metro city and a district town, respectively. Each year 10% of D's population moves to M and 15% of M's population moves to D, where the infrastructure is improving. What is the long-term effect of this migration on the populations of M and of D ? Are they likely to stabilise or not ? Give reasons for your answer. $3\frac{1}{2}$

4. (a) Obtain QR-decomposition for the matrix : 3

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

(b) Let T be a linear operator on C^n, $n > 1$. Prove that the eigenvectors corresponding to distinct eigenvalues of T are linearly independent. 2

5. Which of the following statements are true and which are not ? Give reasons for your answers. Marks will only be given for valid reasons : 10

(i) If λ is an eigenvalue of a linear operator T on $\mathbf{R}^n$, then $T - \lambda I$ is not one-one.

(ii) If A and B are $n \times n$ nilpotent matrices, then A + B is nilpotent.

(iii) Two similar matrices have the same minimal polynomial.

(iv) There is no unitary matrix one of whose column is $\begin{bmatrix} 2 \\ \frac{1}{2} \\ 0 \end{bmatrix}$.

(v) If A is a normal matrix, then A is non-singular.

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