No. of Printed Pages : 5 **MST-003**

POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

02031

Term-End Examination

June, 2019

MST-003 : PROBABILITY THEORY

Time : 3 hours *Maximum Marks : 50*

Note :

*(i) Question no. 1 is **compulsory**.*

*(ii) Attempt any **four** questions from the remaining questions.*

(ii) Use of scientific (non programmable) calculator is allowed.

(iii) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.

(iv) Symbols have their usual meanings.

---

1. State whether the following statements are *True* or *False*. Give reasons in support of your answers. *5×2=10*

   (a) If X and Y are two independent random variables such that X follows $B\left(3, \frac{1}{3}\right)$ and Y follows $B\left(5, \frac{1}{3}\right)$ then $P(X + Y \geq 1) = 0{\cdot}961$.

(b) Each of the two persons, A and B tosses 3 fair coins. The probability that they obtain the same number of heads is $\frac{5}{16}$.

(c) The expectation of the number on an unbiased die when thrown is $\frac{7}{2}$.

(d) If A, B and C are any 3 events such that $P(A) = P(B) = P(C) = \frac{1}{4}$, $P(A \cap B) = P(B \cap C) = 0$ and $P(C \cap A) = \frac{1}{8}$, then the probability that at least one of the events A, B and C occurs is $\frac{7}{8}$.

(e) If the pdf of a random variable is $f(x) = xe^{-x}$, $x \geq 0$, then mean of X is 2.

2. In a company, bolts are manufactured by three machines A, B and C. A turns out twice as many bolts as B and machines B and C produce equal number of bolts. 2% of the bolts produced by each of machines A and B are defective and 4% bolts produced by machine C are defective. All bolts are put into one stock pile and one bolt is chosen from the pile and is found to be defective. What is the probability that the defective bolt is produced by (i) machine A, (ii) machine B and (iii) machine C ? *10*

3. (a) If the random variable X takes the values 1, 2, 3 and 4 such that $3P(X = 2) = P(X = 3) = 5P(X = 4) = 2P(X = 1)$, find the probability distribution of X. *4*

(b) Let a continuous random variable X have pdf $f(x) = kx^2e^{-x}$, $x \geq 0$. Then find (i) value of k and (ii) mean and variance of X. *6*

4. (a) A and B shoot independently until each hits his own target. The probabilities of their hitting the target at each shot are $\frac{3}{5}$ and $\frac{5}{7}$, respectively. Find the probability that B will require more shots than A. *5*

(b) In a basketball championship series between the two teams A and B, the team which wins three games out of five will be the winner. Suppose that team A has probability 0·6 of winning over the team B, then (i) what is the probability that team A will win the series in four games ? (ii) what is the probability that team A will win the series ? *5*

5. (a) The metro trains on a certain route run every 10 minutes between 5 am to 10 pm. What is the probability that a commuter entering a certain station at a random time during this period will have to wait at least 5 minutes ? *5*

(b) In a production of iron rods, the diameter X can be approximated to be normally distributed with mean 2 inches and standard deviation 0·008 inches. What percentage of defectives can we expect if we set the acceptance limits at $2 \pm 0{\cdot}02$ inches ? *5*

6. (a) A box contains 6 green, 5 yellow and 7 white balls. 4 balls are drawn at random from the box. Find the probability that among the balls drawn, there is at least one ball of each colour. *4*

(b) A random variable X has the following probability distribution :

x	− 2	− 1	0	1	2	3
p(x)	0·1	k	0·2	2k	0·3	3k

(i) Find k, (ii) evaluate $P(X < 2)$ and $P(-2 < X < 2)$, and (iii) evaluate the mean of X. *6*

7. (a) Suppose that X has a Poisson distribution. If $P(X = 2) = \frac{2}{3} P(X = 1)$, evaluate (i) $P(X = 0)$ and (ii) $P(X = 3)$. *5*

(b) If on an average, three trucks arrive per hour to be unloaded at a warehouse, using exponential distribution, find the probabilities that the time between the arrival of successive trucks will be (i) less than 5 minutes, and (ii) at least 45 minutes. *5*