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MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc.(MACS)

Term-End Examination, 2019

MMTE-005 : CODING THEORY

Time : 2 Hours] **[Maximum Marks : 50**

(Weightage : 50%)

Note : Answer **any four** questions from questions 1 to 5. Question **6** is **compulsory**. **All** questions carry **equal** marks. Use of calculators is **not** allowed.

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1. (a) Define the following, giving an example of each : [6]

   (i) Self-dual code

   (ii) Hamming weight of a code word

   (iii) Generator Matrix

   (b) Compute the 2-cyclotomic cosets modulo 7. [4]

2. (a) Define a perfect code. Is

$$H = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$ a parity-check

matrix of a perfect code ? Give reasons for your answer. [3]

(b) (i) Construct a parity check matrix of the binary Hamming code H_4 of length 15.

(ii) Using this parity check matrix, decode the vector (001000001100100) and then check that the decoded vector is a code word. [7]

3. (a) Let C be the narrow-sense binary BCH code of designed distance $\delta = 5$, which has a defining set T = {1, 2, 3, 4, 6, 8, 9, 12}. Let α be a primitive 15th root of unity, where $\alpha^4 = 1 + \alpha$, and let the generator polynomial of C be :

$g(x) = 1 + x^4 + x^6 + x^7 + x^8$

If $y(x) = x + x^4 + x^7 + x^8 + x^{11} + x^{12} + x^{13}$ is received, find the transmitted code word. You can use the following table : [6]

0000	0	1000	α^3	1011	α^7	1110	α^{11}
0001	1	0011	α^4	0101	α^8	1111	α^{12}
0010	α	0110	α^5	1010	α^9	1101	α^{13}
0100	α^2	1100	α^6	0111	α^{10}	1001	α^{14}

(b) Let C be the binary code with generator matrix : [4]

$$G = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(i) Is C self-dual ? Justify your answer.

(ii) Find the weight distribution of C.

4. (a) Find all the possible generator polynomials of (3, 1) binary cyclic codes. Find the generator matrix and the parity-check matrix for each code. [4]

(b) Construct the Reed-Muller code G(1, 3). [3]

(c) Prove that if the minimum distance of a code C is d, the minimum distance of the extended code $\hat{C}$ is d or $d+1$. [3]

5. (a) Let C be a cyclic code of length n over F_q with defining set T. Suppose C has minimum weight d. Assume T contains δ-1 consecutive elements for some integer δ. Then show that $\delta \geq 0$. [4]

(b) Let C be the [5, 2] binary code generated by $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{bmatrix}$. Find the weight distribution of C. Find the weight distribution of C^1 by using MacWilliams identity. [6]

6. Which of the following statements are true and which are false ? Justify your answer with a short proof or a counter example : [10]

(a) Every self-orthogonal code is self dual.

(b) The code C = { 00000, 11111} can correct 3 errors.

(c) There is a 2-cyclotomic set modulo 31 of size 7.

(d) The Reed-Muller code R(1, 3) is a self-dual code.

(e) The code C = {0000, 0100, 1000, 0010 } is a cyclic code.

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