1509951

No. of Printed Pages : 4

MMTE-001

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) (MACS)

Term-End Examination

June, 2019

MMTE-001 : GRAPH THEORY

Time : 2 Hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question No. 6 is compulsory. Answer any four from questions 1 to 5. Calculators are not allowed.*

---

1. (a) Prove that a bipartite graph has a unique bipartition if and only if it is connected. 5

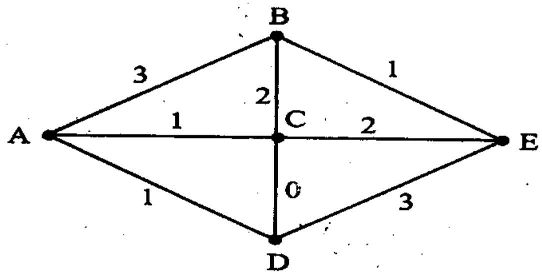
   (b) If G is a simple planar graph with at least 3 vertices, then prove that $e(G) \leq 3n(G) - 6$. Hence decide whether K_6 is a planar graph *or* not. 5

2. (a) If u and v are the only odd-degree vertices in a graph G, then show that G contains a $u - v$ path. 3

(b) State and prove the relationship between the number of vertices and edges in a tree. 5

(c) Show that $K_{1,3}$ is an interval graph. 2

3. (a) Apply Kruskal's algorithm to find a minimal spanning tree in the following graph : 4



(b) (i) Show that for every regular connected X – Y bigraph, | X | = | Y |. 2

(ii) Does every regular connected bipartite graph admit a perfect matching ? Justify your answer. 2

(c) Find the girth of Q_3. 2

4. (a) Construct a graph G for which χ (G) = 4 and ω (G) = 2. Justify your answer. 4

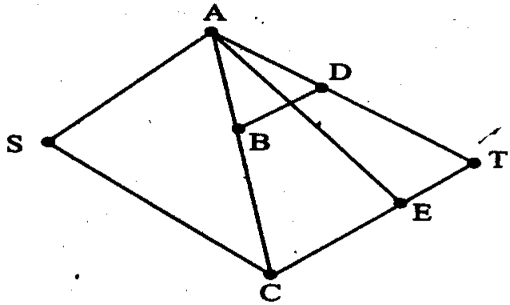
(b) Does every graph G, with $n(G) \geq 2$, have at least *two* vertices of equal degree ? Give reasons for your answer. 3

(c) Give an example, with justification, of two non-isomorphic graphs with the degree sequence 2, 2, 2, 2, 2, 2. 3

5. (a) Draw a tree T with vertices having, eccentricities 4, 4, 4, 3, 3, 2. Justify your answer.

Further, find the central point of T and the radius of T. 4

(b) (i) Apply the breadth first search (bfs) algorithm to the following graph, starting at vertex A and showing the bfs tree at each step.



(ii) Also find the shortest paths from A to every other vertex of this graph. 6

6. Which of the following statements are true ? Give reasons for your answers : 10

(i) There exists a graph with adjacency matrix being the 5×5 zero matrix.

(ii) A graph with n vertices and $n - 1$ edges, for $n \geq 3$, is acyclic.

(iii) Every graph has a perfect matching.

(iv) The chromatic numbers of a planar graph and its dual must be the same.

(v) There is a graph with 6 vertices having 5 cut vertices.