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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination, 2019

MMT-009 : MATHEMATICAL MODELLING

Time : 1½ Hours **Maximum Marks :25**

(Weightage : 70%)

Note : Attempt **any five** questions. Use of non-programmable scientific calculator is allowed.

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1. (a) A patient arrives at the hospital after an overnight fast with a blood glucose concentration of 65 mg/100ml blood . The deviation g(t) of the patient's blood glucose concentration from its optimal concentration satisfies the differential equation [4]

$$\frac{d^2g}{dt^2}+4\alpha\frac{dg}{dt}+(2\alpha)^2g=0,$$ for α a positive constant, immediately after the patient fully absorbs a large amount of glucose. The time t is

measured in hours. If the patient's blood glucose after one hour and two hours are 85mg/100ml and 60mg/100ml of blood respectively, ther find the blood glucose concentration g(t) at the time t. Also find the condition on α for which the patient is normal.

(b) Explain the terms Linear and Non-linear Model giving examples of each. [1]

2. (a) Find a linear demand curve that best fit the following data : [3]

x	20	22	24	26	28	30	32
y	50	55	40	35	30	60	25

(b) The control parameter of growth and decay of a tumour are respectively, 1000 and 500 per day. Also the damaged cells migrate due to vascularization of blood at the rate of 200 cells per day : Find the ratio of the number of tumour cells after 50 days with the initial number of tumour cells. [2]

3. Let $P(W_1, W_2)$ be a portfolio of two securities 1 and 2. Find the values of W_1 and W_2 in the following situations : [5]

(i) ρ_{12} = -1 and P is risk free.

(ii) $\sigma_1 = \sigma_2$ and variance P is minimum.

(iii) Variance P is minimum and ρ_{12} = -0.5, $\sigma_1 = 2, \sigma_2 = 3$

4. Consider the delay model of a population growth given by the difference equation : [5]

u_{n+1}= un exp [(r-u_{n-1})], r > 0

Find the linear steady states of the model and do the stability analysis when $0 < r < \frac{1}{4}$.

5. Consider the following prey-predator model under toxicant stress : [5]

$$\frac{dN_1}{dt} = r_1 N_1 - r_2 C_0 N_1 - b N_1 N_2$$

$$\frac{dN_2}{dt} = -d_1 N_2 + \beta_0 bN_1 N_2$$

$$\frac{dC_0}{dt} = k_1 P - m_1 C_0$$

$$\frac{dP}{dt} = Q - h P - k PN_1$$

Under the conditions

$N_1(0) = N_{10}$, $N_2(0) = N_{20}$, $C_0(0) = 0$, $P(0) = P_0 > 0$

Where $N_1(t)$ = Density of prey population

$N_2(t)$ = Density of predator population

$G(t)$ = Concentration of toxicant in the individual of prey population,

r_1= Growth rate,

d_1= Death rate,

b = predation rate

P_0 = Conversion coefficient

m_1 = depuration rate

k, k_1 = uptake rates

r_2 = death rate due to C_0

Q = rate of toxicant entering into the environment

P = environmental toxicant concentration

Reformulate the above model if the environmental toxicant concentration is assumed to be a constant, equal to α. Do the stability analysis of the reformulated model.

6. (a) A tax consulting firm has 4 service counters in its office for receiving people who have problems and complaints anout their income, wealth and sales taxes. Arrivals average 8o persons in an 8-hours service day. The average service time is 20 minutes, which is found to have an exponential distribution. Calculate the average number of customers in the system and average time a customer spends in the system. [3]

(b) A simple model including the seasonal change that affects the growth rate of a population is given by $\frac{dx}{dt} = C$. x(t) sint, where C is a constant. If x_0 is the initial population, then solve the equation and determine maximum and minimum populations. [2]

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