628681

No. of Printed Pages : 7

MMT-007

M. SC. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE) [M. Sc. (MACS)]

Term-End Examination

June, 2019

MMT-007 : DIFFERENTIAL EQUATIONS AND NUMERICAL SOLUTIONS

Time : 2 Hours *Maximum Marks : 50*

(Weightage : 50%)

***Note :** Question No. 1 is compulsory. Attempt any four questions out of the remaining Question Nos. 2 to 7. Use of calculator is not allowed.*

1. State whether the following statements are True *or* False. Justify your answer with the help of a short proof or a counter example. No marks will be awarded without justification. 2×5

(a) The initial value problem :

$$\frac{dy}{dx} = \frac{y+1}{x^2}, y(0) = 1$$

has an infinity of solutions.

(b) $L\left[\int_0^t (t-x)^2 \sin x \, dx\right] = \dfrac{2}{s(s^2+1)}$, where

L denotes Laplace transform.

(c) In the Crank-Nicolson method, the partial differential equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

is replaced by the finite difference equation :

$(1+r)\, u_1^{j+1} =$

$$u_1^j + \frac{1}{2} r \left(u_{i-1}^{j+1} + u_{i+1}^{j} + u_{i+1}^{j+1} + u_{i-1}^{j} - 2u_i^j \right),$$

where $r = \dfrac{k}{h^2}$ and k and h are mesh lengths in direction of t and x respectively.

(d) For Laguerre polynomials $L_n(x)$:

$$L_4(x) = \frac{1}{\lfloor 4} x^4 - 16x^3 + 36x^2 - 96x + 24.$$

(e) The interval of absolute stability of Runge-Kutta method :

$$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2),$$

$$k_1 = h\, f\,(x_i, y_i)$$

$$k_2 = h\, f\,(x_1 + h, y_i + k_1)$$

is $-2 < \lambda h < 0$.

2. (a) Find the series solution about $x = 0$ of the differential equation : 5

$$x^2 y'' + (x^2 + x)\, y' + (x - 9)\, y = 0.$$

(b) Determine the appropriate Green's function, by using the method of variation of parameters, for the boundary value problem : 5

$$\frac{d^2y}{dx^2} + y = e^{3x} \sin x$$

with $y'(0) = 0$, $y(1) = 0$.

3. (a) Using Laplace transform, solve : 5

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2},\ x > 0,\ t > 0,$$

given that :

$$u(0, t) = 10 \sin 2t,\ u(x, 0) = 0$$

$$u_t(x, 0) = 0,\ \lim_{x \to \infty} u(x, t) = 0.$$

(b) Solve the initial value problem : 5

$$y' = x^2 + \sqrt{y+1}\,,\ y(0) = 1$$

upto $x = 0.4$ using predictor-corrector method :

$$\text{P} : y_{n+1}^{(\text{P})} = y_n + \frac{h}{2}(f_n - f_{n-1})$$

$$\text{C} : y_{n+1}^{(\text{C})} = y_n + \frac{h}{12}[5f(x_{n+1})$$

$$y_{n+1}^{(\text{P})} + 8f_n - f_{n-1}]$$

with step length $h = 0.2$. Compute the starting value using Euler's method and perform two corrector iterations per step.

4. (a) Find the solution of BVP : 6

$$\nabla^2 u = x^2 + y^2,\ 0 \le x \le 1,\ 0 \le y \le 1$$

subject to the boundary condition :

$$u = \frac{1}{12}(x^4 + y^4)$$

on the lines $x = 1$, $y = 0$, $y = 1$ and

$$12u + \frac{\partial u}{\partial x} = x^4 + y^4 + \frac{x^3}{3} \text{ on lines } x = 0$$

using the five-point formula. Assume

$h = \frac{1}{2}$ along both axes. Use central difference approximation in the boundary conditions.

(b) Prove that : 4

$$J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{(\pi x)}}\left[\frac{1}{x}\sin x - \cos x\right]$$

5. (a) Using the Crank-Nicolson method, integrate upto 2 times levels for the solution of initial boundary value problem : 6

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 \le x \le 1,$$

$u(x, 0) = \sin(2\pi x)$

$u(0, t) = 0 = u(1, t)$

with $h = \frac{1}{3}$, $\lambda = \frac{1}{6}$.

(b) Show that : 4

$$\int_{-1}^{+1} x\, P_n(x)\, P_{n-1}(x)\, dx = \frac{2n}{4n^2 - 1},$$

where $P_n(x)$ is the nth degree Legendre polynomial.

6. (a) Use finite Fourier transform to solve : 6

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 4, t > 0$$

Subject to the conditions :

(i) $u(x, 0) = 2x, 0 < x < 4$

(ii) $u(0, t) = 0 = u(4, t), 0 < x < 4, t > 0.$

(b) Obtain the general solution of the differential equation : 4

$$(x+3)^2 \frac{d^2 y}{dx^2} - 4(x+3)\frac{dy}{dx} + 6y = \ln(x+3)$$

7. (a) Find the solution of an initial value problem, subject to given initial and boundary conditions : 5

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$$

$$u(x,0) = 2x \qquad \text{for } x \in \left[0, \frac{1}{2}\right]$$

$$u(x,0) = 2(1-x) \quad \text{for } x \in \left[\frac{1}{2}, 1\right]$$

$u(0, t) = 0 = u(1, t)$

using Schmidt method with $\lambda = \frac{1}{6}$ and $h = 0.2$.

(b) Evaluate : 2

$$L\{t^2 \sin 5t\}.$$

(c) Solve the boundary value problem : 3

$$\frac{d^2y}{dx^2} = y \text{ with } y(1) = 1 \text{ and } y'(0) = 0,$$

using second order finite difference method with $h = \frac{1}{2}$.