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MMT-006

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination, 2019

MMT-006 : FUNCTIONAL ANALYSIS

Time : 2 Hours] **[Maximum Marks : 50**

(Weightage : 70%)

Note : Question **No. 6** is **compulsory**. Attempt **any four** of the remaining questions. Use of calculator is not allowed. Notations as in the study material.

1. (a) Show that the linear map $(x, y) \mapsto (x+y, x-y)$ from $(\mathbb{R}^2, \|\cdot\|_1)$ to $(\mathbb{R}^2, \|\cdot\|_\infty)$ is an isometry. [3]

(b) If A is a bounded normal operator on a Hilbert space H then prove that $\| A^* x \| = \|A x\|$ for all $x \in H$. Prove the converse also. [4]

(c) Suppose M, N are closed subspaces of a Banach space X such that $X = M \oplus N$. Use the closed graph theorem to show that there is a bounded linear map P on X with the properties R(P) = M, N(P) = N and $P^2 = P$. [3]

2. (a) If X,Y are Banach spaces on $X \oplus Y$ define $\|(x,y)\| = \|x\|_X + \|y\|_Y$. Show that this gives a norm making $X \oplus Y$ into a Banach space. [4]

(b) Prove that the dual of c_o is isometric to l^1. [4]

(c) State the principle of Uniform boundedness. [2]

3. (a) If A is a bounded self-adjoint operator on a Hilbert space H, show that $\|A\| = sup\{|<Ax,x>| : x \in H \ \|x\| = 1\}$ [4]

(b) Determine the orthonormal set in l^2 obtained by the Gram-Schmit orthonormalisation process applied to $\{u_n\}$ where $u_n = e_1 + ... + e_n$. [4]

(c) Using Hahn-Banach extension theorem show that a non-zero normed space X has a non-zero dual X^1. [2]

4. (a) Define the spectrum of an operator. Find an operator on C [0,1] whose spectrum is [0,2]. [4]

(b) In L^2 [0,1], determine $\{1, t, t^2, ...\}^{\perp}$. [3]

(c) Consider the shift operator $S : l^2 \to l^2$, given by $(x_1, x_2, ...x_n, ...) \mapsto (0, x_1, x_2, ...x_n, ...)$.
Calculate $\|s^* c_1\|$ and $\|sc_1\|$. [3]

5. (a) Let M be a proper closed subspace of a Hilbert space H and let $x_0 \in H, x_0 \notin M$. Find a bounded linear functional f on H such that $f(x_0) = 1$ and $f(y) = 0$ for all $y \in M$. [3]

(b) Show that l^{∞} is not separable and conclude that l^1 is not reflexive. [4]

(c) If T is a compact operator on a Banach space X, prove that T is also compact when x is given an equivalent norm. [3]

6. State, with justification, whether the following statements are True or False : [5×2=10]

(a) l^2 is a proper subspace of l^3.

(b) Eigen values of a self-adjoint operator are real.

(c) Hahn-Banach extensions on l^1 are unique.

(d) M, N, N_1 are closed subspaces of a Banach space X and $X = M \oplus N = M \oplus N_1$. Then $N = N_1$.

(e) There is a surjective, bounded linear map $T : C \to C_0$.

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