

M. SC. (MATHEMATICS WITH  
APPLICATIONS IN COMPUTER  
SCIENCES) [MSC (MACS)]

Term-End Examination

June, 2019

MMT-005 : COMPLEX ANALYSIS

Time :  $1\frac{1}{2}$  Hours

Maximum Marks : 25

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*Note : Question No. 1 is compulsory. Attempt any  
three questions from Question Nos. 2 to 5.  
Use of calculator is not allowed.*

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1. State giving reasons whether the following statements are true or false : 5×2

(a)  $f(z) = z^{-2}$  is differentiable at  $z = 0$ .

(b) Radius of convergence of the power series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k!} (z - 1 - i)^k \text{ is } \infty.$$

(c)  $\text{Res} \left( \tan z, \frac{\pi}{2} \right) = -1.$

- (d) Every Möbius transformation other than identity has exactly two fixed points.
- (e) For every  $z \in \mathcal{C}$ ,  $\log e^z = z$ .
2. (a) Show that  $u(x, y) = x^3 - 3xy^2 - 5y$  is harmonic in the entire complex plane. Find the harmonic conjugate function of  $u$ . 3
- (b) Show the image of the circle  $|z| = 2$  under the linear fractional transformation  $w = \frac{z+2}{z-1}$  is  $|w-2| = 2$ . 2
3. (a) State and prove Liouville's theorem. 3
- (b) Find the maximum value of the function : 2

$$f(z) = \frac{1}{z+2} \text{ in } |z| \leq 1.$$

4. (a) Find the Laurent series representation of the function :

$$f(z) = \frac{z}{(z-1)^3(z+3)}$$

in the annular region  $|z-1| < 4$ . 3

[3]

(b) Show that  $z = \infty$  is a pole of order 1 of:

$$f(z) = \frac{5z^3}{(z+1)(z-4)}. \quad 2$$

5. Evaluate the integral : 5

$$\int_{-\infty}^{\infty} \frac{x \, dx}{(x^2 - 2x + 2)^2}.$$