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MMT-003

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00781

Term-End Examination

June, 2019

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

Note : *Question no.* ***4*** *is* ***compulsory****. Attempt any* ***four*** *questions from the rest of the questions. Calculators are not allowed.*

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1. (a) If G is a finite group and Z(G) is its centre, prove that $|G| = |Z(G)| + \sum_{i=1}^{n} |C_i|$, where the sum is over all the distinct conjugacy classes containing more than one element. Further, give the class equation of the Klein 4-group. 5

   (b) If χ is a character of a finite-dimensional representation of a finite group G, show that $|\chi(g)|$ is maximum for g = e, the identity element of G. 5

2. (a) Check whether a group of order 156 is simple or not. *3*

(b) Calculate the Legendre symbol $\left(\frac{29}{541}\right)$. Justify each step in the calculation. *4*

(c) Define a 'characteristic subgroup' of a group. Also give an example of a subgroup H, of a group G, which is not a characteristic subgroup of G. Justify your choice of example. *3*

3. (a) Let $\mathbf{R}$ be the set of all real numbers and let $*$ be a binary operation on $\mathbf{R}$, given by $a * b = |a| . b$, for all $a, b \in \mathbf{R}$, where $|a|$ denotes the absolute value of a. Check whether $(\mathbf{R}, *)$ is a semigroup or not. If it is, is it also a monoid ? If $(\mathbf{R}, *)$ is not a semigroup, find its group kernel. Give reasons for your answer. *3*

(b) For any prime p, show that there are no field homomorphisms between $\mathbf{F}_{p^2}$ and $\mathbf{F}_{p^3}$ in either direction. *4*

(c) Give an example, with justification, of a non-trivial irreducible representation of D_4. *3*

4. State, with reasons, which of the following statements are *true* and which are *false*. *10*

(i) The polynomial $x^2 + 2x + 2 \in \mathbf{F}_3[x]$ is irreducible over $\mathbf{F}_3$.

(ii) Any free abelian group is a free group.

(iii) The number of non-isomorphic abelian groups of order 180 is four.

(iv) There is a non-abelian group G for which there exists a faithful representation $\mu : G \to GL_n(F)$ such that $\mu(g)$ is a diagonal matrix for every $g \in G$.

(v) The characteristic of a field extension of $\mathbf{F}_{p^3}(x)$ is 3.

5. (a) Let $P \in SO_3(\mathbf{C})$. Check whether or not 1 is an eigenvalue of P. *4*

(b) Find $[K : \mathbf{Q}]$, where $K = \mathbf{Q}(\sqrt{5}, \sqrt[5]{11})$, giving detailed reasons for your answer. Further, is $X^5 - 11$ irreducible over $\mathbf{Q}(\sqrt{5})$? Why, or why not ? *6*

6. (a) Check whether or not $x^3 + 2x + 1 \in \mathbf{F}_5[x]$ is a primitive polynomial. *6*

(b) Let F, L, K be fields such that K/F is Galois and $F \subseteq L \subseteq K$. Then prove or disprove that : *4*

(i) L/F is Galois.

(ii) K/L is Galois.