No. of Printed Pages : 3

MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

01111

June, 2019

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours — *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no.* **5** *is* ***compulsory****. Answer any* ***three*** *questions from questions no. 1 to 4. Use of calculators is* ***not*** *allowed.*

---

1. Let T be linear operator on $\mathbf{R}^3$.

Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ be a basis of $\mathbf{R}^3$. The matrix of T with respect to $\mathcal{B}$ is $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

Check whether or not T is a bijection. If T^{-1} exists, write down the matrix of T^{-1} with respect to the basis $\mathcal{B}' = \left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$. If T^{-1} does not exist, check whether or not T is diagonalisable. *5*

2. (a) Write the Jordan form of a 4×4 matrix whose minimal polynomial is $(x-3)^2 (x-2)$ and the geometric multiplicity of 3 is two, giving reasons for your answer. *$1\frac{1}{2}$*

   (b) Show the the matrix $B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ is positive semi-definite. Find a positive semi-definite matrix A such that $A^2 = B$. *$3\frac{1}{2}$*

3. (a) Find the least square solution to : *4*

$$x + y + t = 1$$

$$x - y = 2$$

$$x + y = 2$$

$$y + t = 1$$

   (b) If USV^* is the SVD of $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, find S and V. *1*

4. (a) Construct the QR-decomposition for

$$X = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$ *3*

(b) Find a 2×2 matrix X such that $e^A = e^2 X$, where $A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$. *2*

5. Which of the following statements are *True* and which are *False* ? Justify your answers. *5×2=10*

(a) If two $n \times n$ matrices have the same determinant and trace, they must be similar.

(b) A nilpotent matrix has at least one of the entries 0.

(c) The generalized inverse of an invertible matrix is its inverse.

(d) Every unitary matrix has determinant 1.

(e) Every normal operator is self-adjoint.