No. of Printed Pages : 4

BICE-027

B.Tech. – VIEP – MECHANICAL ENGINEERING / B.Tech. CIVIL ENGINEERING (BTMEVI / BTCLEVI)

Term-End Examination

00535

June, 2019

BICE-027 : MATHEMATICS-III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any **two** parts from each question. Use of scientific calculator is permitted. All questions carry equal marks.*

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1. (a) Obtain the Fourier series for the function $f(x) = x^2, -\pi \leq x \leq \pi$. Hence show that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

(b) Obtain Fourier series for

$$f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2 - x), & 1 \leq x \leq 2 \end{cases}.$$

(c) Expand $f(x) = x$ as a half range sine series in $0 < x < 2$. *2×7=14*

2. (a) Solve the partial differential equation :

$(y^2 + z^2)\, p - xyq = - zx$

(b) Solve the linear partial differential equation :

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$$

(c) Solve the partial differential equation :

$(D^2 - D'^2 - 3D + 3D')\, z = xy + e^{x+2y}$ *2×7=14*

3. (a) Solve the P.D.E. by separation of variable method :

$$u_{xx} = u_y + 2u,\ u(0, y) = 0,$$

$$\frac{\partial}{\partial x}\, u(0, y) = 1 + e^{-3y}$$

(b) A tightly stretched flexible string has its ends fixed at x = 0 and x = l. At time t = 0, the string is given a shape defined by $F(x) = \mu x\, (l - x)$, μ is a constant and then released. Find the displacement y(x, t) of any point k of the string at any time t > 0.

(c) The temperature distribution in a bar of length π, which is perfectly insulated at ends $x = 0$ and $x = \pi$ is governed by the partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

Assuming the initial temperature distribution as $u(x, 0) = f(x) = \cos 2x$, find the temperature distribution at any instant of time. *2×7=14*

4. (a) An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. This end is maintained at temperature u_0 at all points and the other edges are at zero temperature. Determine the temperature at any point of the plate in the steady state.

(b) The diameter of a semi-circular plate of radius 'a' is kept at 0°C and the temperature at the semi-circular boundary is T°C. Show that the steady state temperature in the plate is given by

$$u(r, \theta) = \frac{4T}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \left(\frac{r}{a}\right)^{2n-1} \sin (2n-1)\,\theta.$$

(c) Use the method of separation of variables to solve the equation

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \text{ given that } u(x, 0) = 6e^{-3x}.$$

2×7=14

5. (a) Find the Fourier transform of

$$F(t) = \begin{cases} t, & \text{for } |t| < a \\ 0, & \text{for } |t| > a \end{cases}$$

(b) Find the Fourier cosine transform of $\frac{1}{1+x^2}$ and hence find Fourier sine transform of $\frac{x}{1+x^2}$.

(c) Find the Fourier sine transform of $e^{-|x|}$.

Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$. *2×7=14*