BME-001

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

00652

Term-End Examination

June, 2019

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of statistical tables and calculator is permitted.*

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1. Answer any ***five*** of the following : *5×4=20*

(a) Give an example of each of the following, justifying your answer :

(i) A one-one function which is not onto

(ii) Onto function which is not onto

(iii) A function which is both one-one and onto

(iv) A function which is neither one-one nor onto

(b) Evaluate the limits that exist :

(i) $\underset{x \to 0}{\text{Lt}} \dfrac{\sqrt{3+x} - \sqrt{3}}{x}$

(ii) $\underset{x \to -4}{\text{Lt}} \left(\dfrac{3x}{x+4} + \dfrac{8}{x+4} \right)$

(c) Discuss the continuity of the function

$$f(x) = \begin{cases} 2x - 1 & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases} \text{ at } x = 0.$$

(d) Find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$ for the curve

$$\left. \begin{aligned} x &= a \cos t \\ y &= a \sin t \end{aligned} \right\} 0 \leq t \leq \pi.$$

(e) Find two positive numbers such that their sum is 12 and their product is as large as possible.

(f) Evaluate ***one*** of the following integrals :

(i) $\displaystyle\int \frac{\operatorname{cosec}^2 x}{1 + \cot x} \, dx$

(ii) $\displaystyle\int \frac{x^2 + 4}{x^2 + 2x + 3} \, dx$

(g) Find the area of the region enclosed between the curve $x^2 = 4y$ and the line $x = 4y - 2$.

(h) Solve ***one*** of the following differential equations :

(i) $(2x + 3y - 6)\, dy = (6x - 2y - 7)\, dx$

(ii) $(2x - 10y^3) \dfrac{dy}{dx} + y = 0$

2. Answer any ***four*** of the following : *4×5=20*

(a) Find the constant λ so that the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{c} = 3\hat{i} + \lambda\hat{j} + 3\hat{k}$ are coplanar.

(b) Find the directional derivative of $4xz^3 - 3x^2y^2z^2$ at $(2, -1, 2)$ along the y-axis.

(c) The gravitation force $\vec{p}$ of attraction of two particles is the gradient of scalar function $f(x, y, z) = \dfrac{C}{r}$. Show that for $r > 0$, $\vec{p}$ is divergence free.

(d) Find the work done by moving a particle once round the circle $x^2 + y^2 = 9$, if the field of force is given by $\vec{F} = (2x - y - z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$.

(e) Evaluate

$$\oint_C [(x^2 - 2xy)\, dx + (x^2 y + 3)\, dy]$$

around the boundary C of the region $y^2 = 8x, x = 2$.

(f) For any arbitrary vector $\vec{a}$, prove that

$$\hat{i} \times (\vec{a} \times \hat{i}) + \hat{j} \times (\vec{a} \times \hat{j}) + \hat{k} \times (\vec{a} \times \hat{k}) = 2\vec{a}.$$

(g) (i) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that

$\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$, and

(ii) $\text{curl}(\vec{v}) = 0$, if $\vec{v} = \hat{r}$.

3. Answer any ***five*** of the following : *5×3=15*

(a) (i) Does there exist a linear transformation $T : R^3 \to R^2$ such that $T(1, 1, 1) = (2, 3)$ and $T(1, 2, 0) = (1, 2)$ and $T(4, 5, 3) = (5, 10)$?

(ii) Show that every orthogonal set of non-zero vectors is linearly independent.

(b) Find x and y such that

$$\begin{bmatrix} -2 & -1 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 8 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

(c) If a + b + c = 0, solve for x the equation

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0.$$

(d) Solve the following system of equations by matrix method :

$$x + y + z = 1$$

$$x + 2y = 3$$

$$x + 2y + z = 7$$

(e) Find the rank of the matrix

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ -6 & -9 & -12 \end{bmatrix}.$$

(f) Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

and find its inverse.

(g) Show that

$$\begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$$

is a Hermitian matrix.

4. Answer any ***three*** of the following : *3×5=15*

(a) Ten percent of screws produced in a factory turn out to be defective. Find the probability that in a sample of 10 screws chosen at random, exactly two will be defective.

(b) The probability density function of a variable X is

X	0	1	2	3	4	5	6
P(X)	K	3K	5K	7K	9K	11K	13K

What will be the minimum value of K so that $P(X \leq 2) > 3$?

(c) The heights of students of a class are normally distributed. If 11·51% of the students are taller than 70·4 inches and 9·68% are shorter than 65·4 inches, find the mean and standard deviation for the data of heights of students.

(d) In a certain factory producing cycle tyres there is a small chance of 1 in 500 tyres to be defective. The tyres are supplied in lots of 10. Using Poisson distribution, calculate the approximate number of lots containing no defective, one defective and two defective tyres, respectively, in a consignment of 10,000 lots.

(Use $e^{-0 \cdot 02} = 0 \cdot 9802$).

(e) When flipped 1000 times, a coin landed 515 times heads up. Does it support the hypothesis that the coin is unbiased ?

(f) An automatic machine was designed to pack exactly 2 kg of tea. A sample of 100 packs was examined to test the machine. The average weight was found to be 1·94 kg with standard deviation of 0·10 kg. Is the machine working properly ?