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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

00952 **June, 2019**

ET-102 : MATHEMATICS – III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any* ***ten*** *questions. All questions carry equal marks. Use of calculator is allowed.*

1. Test the convergence of the series : 7

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots$$

2. Examine the convergence or divergence of the following series : 7

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \ldots + \frac{2^{n+1} - 2}{2^{n+1} + 1}x^n + \ldots (x > 0)$$

3. Find the Fourier series for the function 7

$$f(x) = x + x^2, \quad -\pi < x < \pi.$$

Hence show that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$$

4. Find the transformation which maps the points $-1, i, 1$ of the z-plane onto $1, i, -1$ of the w-plane respectively. 7

5. Find the Laplace transform of 7

$$e^{-3t}(\cos 4t + 3 \sin 4t).$$

6. Find the Inverse Laplace transform of 7

$$\frac{15}{s^2 + 4s + 13}.$$

7. An electrostatic field in the xy-plane is given by the potential function $\phi = 3x^2y - y^3$. Find the stream function. 7

8. If $w = \phi + i\psi$, represents the complex potential for an electrical field and

$$\psi = x^2 - y^2 + \frac{x}{x^2 + y^2},$$

determine the function ϕ. 7

9. Solve : 7

$$\frac{dy}{dx} + y \cot x = \cos x$$

10. Solve : 7

$$\cos x \frac{dy}{dx} = y \sin x + y^3 \cos^2 x$$

11. Solve : 7

$$\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 9y = e^{3x}$$

12. Solve : 7

$$(x^2 - y^2 - z^2) p + 2xyq = 2xz$$

13. Determine the poles of the function

$$f(z) = \frac{z^2}{(z-1)^2 (z+2)} \text{ and}$$

the residue at each pole. 7

14. If

$$x + \frac{1}{x} = 2 \cos \theta, \text{ and } y + \frac{1}{y} = 2 \cos \phi,$$

then prove that one of the values of

$$\frac{x^m}{y^n} + \frac{y^n}{x^m} \text{ is } 2 \cos (m\theta - n\phi).$$ 7

15. Apply the Hurwitz – Routh criterion to determine the stability of the systems whose characteristic equation is given by 7

$$s^4 + 5s^3 + 2s + 10 = 0.$$

16. For the following system, we assume that the weight (w) is guided so that only vertical motion, without swinging is possible. Obtain the differential equation representing this system. 7