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ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

00952

June, 2019

ET-101(A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of scientific calculator is allowed.*

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1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate :

$$\lim_{x \to 2} \frac{x^2 - 3x + 2}{x^2 - 6x + 8}$$

(b) If $\sin y = x \sin(a + y)$, prove that

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}.$$

(c) Find the equation of the tangent and normal to the curve $y = x^3 - 3x^2 - x + 5$ at the point where $x = 3$.

(d) Calculate the radius and the height of a right circular cylinder of maximum volume which can be cut from a sphere of radius R.

(e) Verify Rolle's theorem for the function

$$f(x) = x^3 + 5x^2 - 6x,$$

on the interval (0, 1).

(f) If $x = r \cos \theta$, $y = r \sin \theta$;

$$\text{evaluate } \frac{\partial(x, y)}{\partial(r, \theta)}, \text{ and } \frac{\partial(r, \theta)}{\partial(x, y)}.$$

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate any ***one*** of the following :

(i) $\displaystyle\int \frac{1}{16 + x^2}\, dx$

(ii) $\displaystyle\int \frac{\cos x - \cos 2x}{1 - \cos x}\, dx$

(b) Evaluate any ***one*** of the following :

(i) $\displaystyle\int_1^{\sqrt{3}} \frac{1}{1 + x^2}\, dx$

(ii) $\displaystyle\int \sin^5 x \cos^4 x\, dx$

(c) Evaluate :

$$\int_1^2 x^2 \log x\, dx$$

(d) Find the area bounded by the x-axis and the curve $y = 1 - x^2$.

(e) Prove that

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx = \frac{\pi}{4}.$$

(f) The velocity of a train which starts from rest is given by the following table, the time being reckoned in minutes from the start and speed in kilometres per hour.

Time (in minutes)	Speed (km/hr.)
2	10
4	18
6	25
8	29
10	32
12	20
14	11
16	5
18	2
20	0

Estimate approximately by Simpson's $\frac{1}{3}$ rd rule, the total distance covered in 20 minutes.

3. Answer any ***four*** of the following : *4×4=16*

(a) Show that the vector $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the sides of a right angled-triangle.

(b) If $\vec{A} = 3\hat{i} - \hat{j} - 4\hat{k}$, $\vec{B} = -2\hat{i} + 4\hat{j} - 3\hat{k}$, and $\vec{C} = \hat{i} + 2\hat{j} - \hat{k}$, find the unit vector parallel to $3\vec{A} - 2\vec{B} + 4\vec{C}$.

(c) A particle acted upon by constant forces $2\hat{i} + \hat{j} - \hat{k}$, $\hat{i} - 2\hat{j} + 3\hat{k}$, and $3\hat{i} + \hat{j} + 5\hat{k}$ is displaced from the point $\hat{i} + 2\hat{j} + 3\hat{k}$ to point $6\hat{i} + 3\hat{j} + \hat{k}$. Find the work done.

(d) Find grad ϕ when ϕ is given by

$$\phi = 3x^2 y - y^3 z^2$$

at the point (1, – 2, – 1).

(e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$,

show that

(i) div $\vec{r} = 3$, and

(ii) curl $\vec{r} = \vec{0}$.

(f) Show that the vector

$$\vec{V} = (x + 3y)\hat{i} + (y - 3z)\hat{j} + (x - 2z)\hat{k}$$

is solenoidal.

4. Answer any *six* of the following : *6×3=18*

(a) Find x, y, z and w if

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}.$$

(b) If $A + B = \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}$ and $A - B = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$,

calculate the product AB.

(c) If

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}, \text{ find } A^{-1}.$$

(d) Show that

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}^{-1}$$

(e) Find the rank of the following matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

(f) Solve the following system of equations by Cramer's rule.

$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8$$

(g) If

$$A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix},$$

show that A is a Hermitian matrix.

(h) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$
