# MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS 

Time : 3 Hours]

Note : Question number 1 is compulsory. Attempt any three questions from the rest.

1. (a) Multiply the following two matrices using Strassen's algorithm :

$$
\left[\begin{array}{cc}
5 & 6 \\
-4 & 3
\end{array}\right] \text { and }\left[\begin{array}{cc}
-7 & 6 \\
5 & 9
\end{array}\right]
$$

(b) Explain Quick sort algorithm using suitable example.
(c) Prove that running time of $T(n)=n^{3}+20 n+1$ is $\mathrm{O}\left(\mathrm{n}^{3}\right)$.
(d) Explain Push Down Automata (PDA) with suitable examples.
(e) Define fractional Knap sack problem and give a greedy algorithm to solve this problem efficiently.
(f) Find the topological ordering of the following graph :

(g) Consider the following Context Free Grammar (CFG) :

$$
\binom{S \leftarrow S S|X a X a X|^{\wedge}}{\left.X \rightarrow b X\right|^{\wedge}}
$$

Explain the language generated by CFG.
(h) What is an algorithm ? Explain characteristics of an algorithm with the help of an example.
2. (a) What is Minimum Spanning tree ? Write Prim's algorithm for finding minimum spanning tree and find its time complexity. Also find MST of the following graph using Prim's algorithm :

(b) Define Halting Problem of Turing Machine with an example.
(c) Show that there does not exist algorithm for deciding whether or not $L\left(G_{A}\right) \cap L\left(G_{B}\right)=\phi$ for arbitrary context free grammars $G_{A}$ and $G_{B}$. [5]
3. (a) Prove that running time of binary search algorithm in worst case is $O\left(\log _{2} n\right)$.
(b) Explain how 0|1 Knapsack problem can be solved using Dynamic Programming.
(c) What is "Principle of optimality" in Dynamic programming ? Explain how dynamic programming can be used to solve a chain of Matrix Multiplication. Apply Dynamic Programming to multiply the following fair Matrices:

$$
\begin{aligned}
& \left\langle M_{1}, M_{2}, M_{3}, M_{4}\right\rangle \text { with dimensions } \\
& \langle(15,3),(3,8) ;(8,9),(9,7)\rangle
\end{aligned}
$$

4. 

(a) Differentiate between class P, NP and NPcomplete problems.
(b) Show a polynomial time reduction from the clique problem to the vertex cover problem by giving an example.
[9]
(c) Write Euclid's algorithm for finding Greatest Common Qivisor (GCD) of two natural numbers $M$ and $N$.
5. (a) Represent the following graph using (i) Array; and (ii) Adjacency list
[6]

(b) Trace how Depth First Search Traverses the followíng Graph when starting at node A :

(c) If $L_{1}$ and $L_{2}$ are Context Free Language (CFL), Prove that $L_{1} \cup L_{2}$ is also Context Free.

