

**POST GRADUATE DIPLOMA IN
APPLIED STATISTICS (PGDAST)**

Term-End Examination

01465

June, 2018

MST-003 : PROBABILITY THEORY

Time : 3 hours

Maximum Marks : 50

Note :

- (i) *Attempt all questions. Questions no. 2 to 5 have internal choices.*
- (ii) *Use of scientific calculator is allowed.*
- (iii) *Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.*
- (iv) *Symbols have their usual meanings.*

1. State whether the following statements are *True* or *False*. Give reasons in support of your answers. $5 \times 2 = 10$

- (a) If three unbiased coins are tossed simultaneously, then the probability of getting two heads is $\frac{3}{8}$.

- (b) If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B) - P(A)P(B)$.
- (c) If a discrete random variable X has the probability distribution as

X	0	1	2
p(x)	c	2c	3c

then c must be 8.

- (d) If a random variable X follows Poisson distribution such that $P(X = 1) = P(X = 2)$, then mean of the distribution is 2.
- (e) If X is uniformly distributed over $(-5, 5)$, then $P(X > 4)$ will be 0.1.
2. (a) Three athletes A, B and C are participating in the Olympics. A is twice as likely to win as B and B is twice as likely to win as C. What are the probabilities of their winning? 5
- (b) The odds in favour of passing a driving test by a person A are 3 : 5 and odds in favour of passing the same test by another person B are 3 : 2. What is the probability that at least one of them will pass the test? 5

OR

(a) In an office, three clerks are assigned to process incoming mail. The first clerk processes 40%, the second clerk processes 35% and the third clerk processes 25% of the mail. The first clerk has an error rate of 0.04, the second has an error rate of 0.06 and the third has an error rate of 0.03. A mail selected at random from a days output is found to have an error. The manager of the office wishes to know the probability that the mail was processed by the first, second or third clerk. Find the desired probabilities. 8

(b) Past experience of 200 consecutive days speaks that weather forecast of a station is 120 times correct. A day of the year is selected at random. Find the probability that weather forecast on this day is false. 2

3. (a) The joint probability distribution of a pair of random variables (X, Y) is given in the following table : 2+3

X Y	-1	0	1
-1	0	0	$\frac{1}{3}$
0	0	0	0
1	0	$\frac{1}{3}$	$\frac{1}{3}$

Find :

- (i) Marginal distributions of X and Y .
 - (ii) Conditional distribution of X given $Y = 1$.
- (b) The diameter of an electric cable, say X , is assumed to be a continuous random variable with p.d.f. 1+4

$$f(x) = 6x(1 - x); 0 \leq x \leq 1.$$

- (i) Check that $f(x)$ is a p.d.f.
- (ii) Determine $P(X \leq 0.5)$ and $P(X > 0.2)$.

OR

- (a) A player tosses two unbiased coins. He wins ₹ 10 if 2 heads appear, ₹ 5 if one head appears and loses ₹ 1 if no head appears. Find the expected value of the amount won by him. 4

- (b) If (X, Y) be two-dimensional random variables having joint density function,

$$f(x, y) = \begin{cases} 2; & 0 < x < 1 \text{ and } 0 < y < x \\ 0; & \text{otherwise} \end{cases}$$

- (i) Find the marginal density functions of X and Y .
(ii) Check for independence of X and Y . 4+2

4. (a) The chances of catching cold by workers working in an ice factory during winter are 30%. What is the probability that out of 6 workers, 4 or more will catch cold? 5

- (b) If the probability that an individual suffers from a bad reaction by an injection of a given serum is 0.002, determine the probability that out of 500 individuals,

- (i) exactly 3, and
(ii) more than 1

individuals suffer from bad reaction. 2+3

OR

(a) A box of 25 cricket balls contains 10 defective balls. A quality control inspector inspects 2 randomly selected balls from the box. She accepts the box if at most one ball is found defective, otherwise all the remaining balls are inspected. Find the probability that the box is accepted without further inspection. 5

(b) An unbiased die is cast until 6 appears. What is the probability that it must be cast more than 6 times ? 5

5. An aptitude test was conducted on 900 employees of a company. The mean score was found to be 50 with SD of 20. On the basis of this information, find the number of employees whose

(i) mean score was less than 30,

(ii) mean score exceeded 70, and

(iii) mean score was between 30 and 70. 4+3+3

OR

(a) Metro trains are scheduled every 3 minutes at a certain station. A person comes to the station at a random time and waits for the train. If waiting time follows uniform distribution over the interval $(0, 3)$, then find the probability that he has to wait at least 2 minutes for the train. 4

(b) Suppose that on an average 1 customer per minute arrives at a shop. What is the probability that the shopkeeper will wait more than 3 minutes before the first two customers arrive? Assume that waiting time follows gamma distribution. 6
