No. of Printed Pages : 4

MMTE-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

Term-End Examination

00245

June, 2018

MMTE-005 : CODING THEORY

Time : 2 hours | *Maximum Marks : 50*

(Weightage : 50%)

Note : *Answer any **four** questions from questions no. 1 to 5. Question no. **6** is **compulsory**. Calculators are not allowed.*

---

1. (a) Give an example of a [7, 4] linear code, with justification. Further, give a generator and a parity-check matrix of this code. *6*

(b) Write the generator matrix of the binary [7, 4] cyclic code with generator polynomial $(x^3 + x + 1)$. Also find the parity check matrix of this code. *4*

2. (a) Prove that the integers modulo n do not form a field if n is not prime. *2*

(b) The systematic generator matrix for a [6, 3] linear block code is

$$G = \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Find the standard array for syndrome decoding. *5*

(c) Check whether the polynomial $x^3 + x + 1 \in \mathbf{F}_{32}[x]$ is primitive. You are given that $x^{30} \bmod (x^3 + x + 1) = x^2 + 1$. You may assume that the polynomial is irreducible. *3*

3. (a) Find a generator polynomial for a [13, 10] – BCH code. Use $x^3 + 2x + 1$ as the primitive polynomial for $\mathbf{F}_{27}$, and the table below : *5*

i	α^i	i	α^i
1	α	14	2α
2	α^2	15	$2\alpha^2$
3	$\alpha + 2$	16	$2\alpha + 1$
4	$\alpha^2 + 2\alpha$	17	$2\alpha^2 + \alpha$
5	$2\alpha^2 + \alpha + 2$	18	$\alpha^2 + 2\alpha + 1$
6	$\alpha^2 + \alpha + 1$	19	$2\alpha^2 + 2\alpha + 2$
7	$\alpha^2 + 2\alpha + 2$	20	$2\alpha^2 + \alpha + 1$
8	$2\alpha^2 + 2$	21	$\alpha^2 + 1$
9	$\alpha + 1$	22	$2\alpha + 2$
10	$\alpha^2 + \alpha$	23	$2\alpha^2 + 2\alpha$
11	$\alpha^2 + \alpha + 2$	24	$2\alpha^2 + 2\alpha + 1$
12	$\alpha^2 + 2$	25	$2\alpha^2 + 1$
13	2		

(b) Find the weight distribution of a binary code generated by

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Find the weight enumerator polynomial of the code. Also, find the weight enumerator polynomial of the dual code. *5*

4. (a) Let C be any $\left[n, \frac{n-1}{2}\right]$ cycle code over $\mathbf{F}_q$. Prove that C is self-orthogonal if and only if C is an even-like duadic code whose splitting is given by μ_{-1}. 5

(b) List all codewords of the code C over $\mathbf{Z}_4$ generated by $\begin{bmatrix} 1 & 2 & 3 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \end{bmatrix}$. Also find the Lee weight distribution of the code. 5

5. (a) Let C_2 be the (4, 2) convolutional code, with generator matrix

$$G_2 = \begin{bmatrix} 1 & 1+D+D^2 & 1+D^2 & 1+D \\ 0 & 1+D & D & 1 \end{bmatrix}$$

If G_2 is used to encode the message (11010, 10111) using C_2, what is the codeword ? Find the four component functions used in encoding. What is the memory of the code ? 5

(b) Let C be the [7, 4, 2] binary code with the following parity-check matrix

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(i) Give the Tanner graph for this code.

(ii) List all the codewords of the dual code of C and hence, find its minimum distance. 5

6. Which of the following statements are *True* and which are *False* ? Give reasons for your answers. Marks will only be given for valid reasons. *10*

(a) $2 + x + x^2 + x^3$ is irreducible in $\mathbf{Z}_3[x]$.

(b) Given

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix},$$

there is a unique codeword of weight 3 corresponding to H, where H is the parity check matrix of a binary code.

(c) A quadratic residue code of length 7 exists over $\mathbf{F}_3$.

(d) The parity check matrix of a turbo code can be the identity matrix.

(e) Every perfect code is a self dual code.