No. of Printed Pages : 4

MMTE-001

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00075

Term-End Examination

June, 2018

MMTE-001 : GRAPH THEORY

Time : 2 hours *Maximum Marks : 50*

(Weightage : 50%)

Note : *Question no. 1 is* ***compulsory****. Answer any* ***four*** *from questions* ***2*** *to* ***6****. Electronic computing devices are not allowed. Draw diagrams wherever necessary.*

1. State whether *true* or *false*, with suitable justification in the form of a short proof or a counter example. *5×2=10*

(a) A graph with n vertices and n – 1 edges is a tree.

(b) There are graphs G with diam G = rad G.

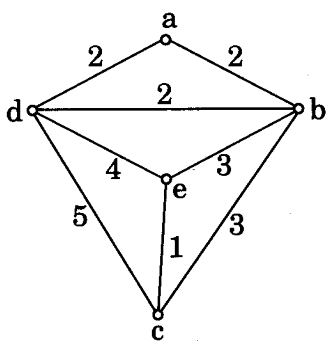
(c) A simple connected graph G with $e(G) \leq 3n(G) - 6$ is always planar, where $e(G)$ denotes the number of edges in G.

(d) If G is a k-regular bipartite graph, $k \geq 1$, with bipartition X, Y, then $|X| = |Y|$.

(e) Every graph with at least five vertices is four-colourable.

2. (a) Let G be a simple graph having no isolated vertex and no induced subgraph with exactly two edges. Show that G is a complete graph. *6*

(b) Find a minimum spanning tree of the following graph using Kruskal's algorithm : *4*



3. (a) If every vertex of a graph has a degree of least two, then show that G contains a cycle. *3*

(b) Draw the Petersen graph. Check whether it is Eulerian or not. Show that it is not planar. 7

4. (a) Prove that an integer list d of size $n > 1$ is graphic if and only if the list d' is graphic, where d' is obtained from d by deleting its largest element Δ and subtracting 1 from the next Δ largest elements. 7

(b) If f is a feasible flow and [S, T] is a source/sink cut, then $val(f) < cap(S, T)$. 3

5. (a) If G is a self-complementary graph with n vertices, show that $n = 4k$ or $4k + 1$ for sum $k \geq 1$. Draw a self-complementary graph with five vertices. 4

(b) Prove that every component of the symmetric difference of two matchings in a graph is a path or an even cycle. 3

(c) Draw a graph G such that $\kappa(G) < \kappa'(G) < \delta(G)$ where κ, κ' and δ denote vertex-connectivity, edge-connectivity and minimum degree. 3

6. (a) Prove that $\chi(G \square H) = \max\{\chi(G), \chi(H)\}$ where χ denotes the chromatic number and $\square$ represents the operation of taking Cartesian product of graphs. 4

(b) Draw a plane embedding of K_4 and its dual. 3

(c) Determine, with justification, whether the graph below is Hamiltonian. 3