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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

00055

Term-End Examination

June, 2018

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Attempt any **five** questions. Use of scientific non-programmable calculator is allowed.*

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1. (a) Classify the following models into linear and non-linear models. Justify your classification. 2

(i) Tumour growth model in a closed region.

(ii) Single species, one-dimensional advection-diffusion-reaction model for the exponential growth rate function.

(b) The growth of a population is proportional to the population and restricted by the availability of food, space, etc., which can be modelled as proportional to the square of the population itself.

(i) Model this process.

(ii) Solve the resulting equation.

(iii) Find the long-term behaviour of the population. *3*

2. A company has factories at F_1, F_2 and F_3 that supply products to warehouses at W_1, W_2 and W_3. The weekly capacities of the factories are 200, 160 and 90 units, respectively. The weekly warehouse requirements are 180, 120 and 150 units, respectively. The unit shipping costs (in ₹) are as follows :

Warehouse

		W_1	W_2	W_3	Supply
Factory	F_1	16	20	12	200
	F_2	14	8	18	160
	F_3	26	24	16	90
	Demand	180	120	150	450

Determine the optimal distribution for this company in order to minimize its total shipping cost. *5*

3. (a) Return distribution of two securities are given below :

Return		Probabilities
X	Y	$p_{xj} = p_{yj} = p_j$
0·16	0·14	0·33
0·12	0·08	0·25
0·08	0·05	0·17
0·11	0·09	0·25

Find which security is more risky in the Markowitz sense. $2\frac{1}{2}$

(b) Formulate the model for which the reproductive function of the cancer cells in the tumour surface is given by $\phi(c) = \dfrac{3 - 2c}{2(1 - 2c)}$; $c \neq \dfrac{1}{2}$ together with initial conditions $c = 20 \times 10^5$ at $t = 0$. Also find the density of the cancer cells in the tumour's surface area at t = 20 days. $2\frac{1}{2}$

4. Do the stability analysis of the trivial equilibrium solution of the following competing species model :

$$\frac{\partial N_1}{\partial t} = a_1 N_1 - b_1 N_1 N_2 + D_1 \frac{\partial^2 N_1}{\partial x^2}$$

$$\frac{\partial N_2}{\partial t} = - d_1 N_2 + c_1 N_1 N_2 + D_2 \frac{\partial^2 N_2}{\partial x^2},$$

$$0 \leq x \leq L,$$

where D_1 and D_2 are diffusion coefficients of the two population densities N_1 and N_2, respectively. a_1 is the growth rate, b_1 is the predation rate, d_1 is the death rate and c_1 is the conversion rate. The initial boundary conditions are

$$N_i(x, 0) = f_i(x) > 0, \ 0 \le x \le L, \ i = 1, 2$$

$$N_i = \overline{N}_i \text{ at } x = 0 \text{ and } x = L \ \forall \ t, i = 1, 2$$

where $\overline{N}_i$ are the equilibrium solutions of the given system of equations. 5

5. Consider the data showing observations on the quantity demanded of a certain commodity depending on commodity price and consumers' income :

Quantity demanded	Price (in ₹)	Income (in ₹)
100	5	1000
75	7	600
80	6	1200
70	6	500
50	8	300
65	7	400
90	5	1300
100	4	1100
110	3	1300
60	9	300

Find a multiple regression equation that best fits the data. 5

6. Consider the budworm population dynamics governed by the equation

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - x,$$

where k, the carrying capacity, and r, the birth rate of the budworm population, are positive parameters. Find out the steady states and use the perturbation to do the stability analysis of the equation for $0 < r < 1$. *5*
