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MMT-005

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

00555

June, 2018

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

Note : *Question no. **1** is **compulsory**. Attempt any **three** questions from questions no. **2** to **5**. Use of calculator is **not** allowed.*

1. State, giving reasons, whether the following statements are *True* or *False* : *5×2=10*

(a) The function

$$f(z) = \begin{cases} \dfrac{\bar{z}^2}{z}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

is differentiable at $z = 0$.

(b) If the series

$$\sum_{n=0}^{\infty} a_n z^n$$

has radius of convergence R_1 and

$$\sum_{n=0}^{\infty} b_n z^n$$

has radius of convergence R_2, then

$$\sum (a_n + b_n) z^n$$

has radius of convergence at least min (R_1, R_2).

(c) The identity $\log (z_1 z_2) = \log (z_1) + \log (z_2)$ is always valid where log is the principal branch of logarithm.

(d) $\int_C \bar{z}\, dz = 0$ where C is upper semi-circular path from -1 to 1.

(e) The transformation $w = \frac{1}{z-1}$ maps the circle $|z| = 1$ to a circle.

2. (a) If $f(z) = u + iv$ is analytic in $\mathbf{C}$ and $u^2 + v^2 = 2(u + v)$, then show that f is constant. 3

(b) Show that

$$\sum_{n=0}^{\infty} \frac{(1-i)^n}{2^n} = 1 - i.$$ 2

3. (a) Find the harmonic conjugate of the function $u(x, y) = xy^3 - x^3y$. 2

(b) Show that

$$\left| \int_C \frac{1}{z^2 + 2} dz \right| \leq 1$$

where C is the straight-line segment from 0 to i. 3

4. (a) Expand $f(z) = \frac{1}{(z-1)^2(z-3)}$ in a Laurent series valid for the annular domain $0 < |z - 1| < 2$. 3

(b) Find the singularities of the function

$$f(z) = \frac{e^{\frac{1}{z}}}{(z-1)^2}$$

and classify them. 2

5. Using contour integration, evaluate

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+9)}$$ 5