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MMT-003

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

Term-End Examination

June, 2018

00405

MMT-003 : ALGEBRA

Time : 2 hours *Maximum Marks : 50*

Note : *Question no.* ***6*** *is* ***compulsory****. Attempt any* ***four*** *questions from questions no. 1 to 5. Calculators are* ***not*** *allowed.*

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1. (a) Let α be a complex root of $x^3 + 2x + 6$. Find the inverse of $\alpha^2 + 2\alpha + 1$ in the form $d + b\alpha + c\alpha^2$, $d, b, c \in \mathbf{Q}$. *5*

(b) Find the number of non-isomorphic abelian groups of order 160. How many of these groups will be non-cyclic, and why ? *5*

2. (a) Let

$$S = \left\{ \begin{bmatrix} A & I \\ 0 & B \end{bmatrix} \middle| A, B \in GL_n(\mathbf{R}), B^t = B, A = (B^{-1}) \right\}$$

Check whether or not $S \subseteq SP_{2n}(\mathbf{R})$. Also give an example of a 6×6 matrix which is in $SP_6(\mathbf{R}) \cap SL_6(\mathbf{R})$. 5

(b) Check whether or not the polynomial $x^2 - x + 2 \in \mathbf{F}_3[x]$ is irreducible. Let β be a root of this polynomial. Is $\mathbf{F}_9 \simeq \mathbf{F}_3[\beta]$? Further, is $\phi : \mathbf{F}_3[\beta] \to \mathbf{F}_3[\beta]$ given by $\phi(\beta) = 1 - \beta$ extend to a field automorphism ? Justify your answers. 5

3. (a) Let $A = \{1, 2, 3, 4, 5\}$ and $S = A^3$. Define an action $o : S_5 \times S \to S$ by $\sigma o(a_1, a_2, a_3) = (\sigma(a_1), \sigma(a_2), \sigma(a_3))$, $\sigma \in S_5$, $(a_1, a_2, a_3) \in S$. 5

(i) Is this action transitive ? Justify your answer.

(ii) Find the stabiliser of $(1, 2, 3) \in S$.

(b) Let α be the positive real fourth root of 3. Factor the polynomial $x^4 - 3$ into irreducible factors in each of the fields $\mathbf{Q}$, $\mathbf{Q}(\alpha)$ and $\mathbf{Q}(\alpha, i)$. 5

4. (a) Consider the group

$$D_4 = \langle x, y; x^4, y^2, yxyx \rangle.$$

Check that the map $\rho : D_4 \to GL_3(\mathbf{R})$, defined by

$$\rho(x) = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\rho(y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

extends to a representation of D_4. Also determine the character of ρ. *5*

(b) Check whether or not a group of order 407 is simple. *3*

(c) Check whether 8173194602 is a valid ISBN number or not. *2*

5. (a) In supermarkets and airports there are automatic door openers with a sensor pad on both sides of the door. As a person approaches the door from either side, the door opens and its remains open till the person goes through the door. The door stays closed only if there is nobody on either side of it. Define a semi-automaton for this situation. *4*

(b) Consider the design given by

$X = \{1, 2, 3, 4, 5, 6, 7\}$,

$B = \{S \subset X \mid |S| = 4\}$.

Find the value of λ for i = 4, 3, 2. *2*

(c) Let C be a binary code with generating matrix

$$G = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

Find the parity check matrix of C. *2*

(d) Write down all the conjugacy classes of A_4. *2*

6. Which of the following statements are *True* and which are *False* ? Justify your answers with a short proof or a counter-example. *10*

(a) There is no group with class equation $1 + 1 + 1 + 3 + 3$.

(b) If a finite group G has a conjugacy class of order d, then G has an irreducible representation of dimension d.

(c) The only elements of order 6 in S_7 are 6-cycles.

(d) If $f(x) = a_0 + a_1x + \ldots + a_{n-1}x^{n-1} + x^n \in \mathbf{Z}[x]$ is irreducible over $\mathbf{Q}[x]$, then the polynomial $\bar{f}(x) = \bar{a}_0 + \bar{a}_1 x + \ldots + \bar{a}_{n-1} x^{n-1} + x^n$ is irreducible over $\mathbf{Z}_p[x]$, where p is a prime and $\bar{a}_i$ denotes the residue class of a_i modulo p, $0 \le i \le n - 1$.

(e) The order of $G(\mathbf{Q}(\sqrt{2})/\mathbf{Q})$ is 1.