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MMT-002

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

M.Sc. (MACS)

00935

Term-End Examination

June, 2018

MMT-002 : LINEAR ALGEBRA

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Question no. 5 is **compulsory**. Answer any **three** questions from questions no. **1** to **4**. Use of calculators is **not** allowed.*

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1. (a) Let the matrix of a linear operator T on $\mathbf{R}^3$ with respect to the standard basis be

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}.$$

Find the matrix of T with respect to the basis

$$\left\{\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}.$$ 2

(b) Write the spectral decomposition of the matrix

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$ 3

2. (a) Prove that the matrix

$$A = \begin{bmatrix} 1 & 3 & 3 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

is diagonalizable. Find an invertible matrix P so that

$$P^{-1} AP = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ $2\frac{1}{2}$

(b) Write a unitary matrix whose first column is

$$\frac{1}{\sqrt{3}} \begin{bmatrix} i \\ i \\ 1 \end{bmatrix}.$$

Check whether the matrix you get is unitarily diagonalizable or not. $2\frac{1}{2}$

3. (a) Write all possible Jordan canonical forms for a 5×5 matrix whose minimal polynomial is $(x-1)(x-2)(x-3)$ and the determinant is 12. *2*

(b) Check whether or not the matrix

$$\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{bmatrix}$$

is positive definite. *2*

(c) Prove that, for any square matrix A, A and A + I are not similar. *1*

4. Write the singular value decomposition of the matrix

$$\begin{bmatrix} -1 & 2 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}.$$ *5*

5. Which of the following statements are *True*, and which are *False* ? Give reasons for your answers. *10*

(a) If T is a linear operator on a finite-dimensional vector space whose matrix with respect to some basis is the identity matrix, then the matrix of T is the identity matrix with respect to any basis.

(b) If A and B are $n \times n$ non-zero matrices, such that AB is a diagonal matrix, then both A and B must be diagonal matrices.

(c) If two 4×4 matrices A and B have the same minimal polynomial, then A and B are similar.

(d) There is a real orthogonal matrix with one of the eigenvalues equal to -2.

(e) $[1, 2]$ is a generalised inverse of $[2, 1]^t$.