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**BME-015**

# B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

00963 **Term-End Examination**

**June, 2018**

## BME-015 : ENGINEERING MATHEMATICS – II

*Time : 3 hours* *Maximum Marks : 70*

***Note :*** *Answer any* ***ten*** *questions. All questions carry equal marks. Use of scientific calculator is permitted.*

**1.** Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 < x < \pi, \ y > 0,$$

subject to the boundary conditions

$u(0, y) = u(\pi, y) = 0$, $u(x, 0) = 1$, and $u(x, y) \to 0$ as $y \to \infty$. 7

**2.** An elastic string of length 20 cm, fixed at both ends is displaced from its position of equilibrium by imparting to each of its points an initial velocity given by

$$g(x) = \begin{cases} x & 0 \le x \le 10 \\ 20 - x & 10 < x \le 20 \end{cases}$$

x being the distance from one end. Determine the displacement function at any time t. 7

**3.** Solve : 7

$$(D^2 - 2DD' + 15D'^2)\, z = 12xy$$

**4.** Solve : 7

$$(D^2 - 4DD' + 4D'^2)\, z = e^{2x+y}$$

**5.** Find the general solution of the partial differential equation 7

$$(x^2 - yz)\, p + (y^2 - zx)\, q = z^2 - xy$$

**6.** Find the Fourier series expansion of the function

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi}, & -\pi < x < 0 \\ 1 - \dfrac{2x}{\pi}, & 0 \le x < \pi \end{cases}$$

where $f(x + 2\pi) = f(x)$.

Also deduce that

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}.$$ 7

**7.** Find the Fourier series for the function

$$f(x) = e^{-x},\ 0 < x < 2\pi,$$ 7

where $f(x + 2\pi) = f(x)$.

8. Find the residue of the following functions at each pole.

$$\frac{z^2 - 2z}{(z+1)^2 (z^2+1)}$$ 7

9. Test for convergence or divergence of the series 7

$$\sum_{n=0}^{\infty} \frac{(100 + 75i)^n}{n!}$$

10. Test the following series for convergence

$$\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \frac{x^4}{4.5} + \ldots \infty \ (x > 0).$$ 7

11. If $2 \cos \theta = x + \frac{1}{x}$, and $2 \cos \phi = y + \frac{1}{y}$,

show that one of the values of

$$\frac{x^m}{y^n} + \frac{y^n}{x^m} \text{ is } 2 \cos (m\theta - n\phi).$$ 7

12. Find the bilinear transformation which maps the points $z = 0, -1, \infty$ into the points $w = -1, -2 - i, i$. 7

**13.** If $w = \phi + i\,\psi$ represents the complex potential for an electric field and $\psi = x^2 - y^2 + \frac{x}{x^2 + y^2}$, determine the function $\phi$. 7

**14.** Prove that

$$\int_C (z-a)^n \, dz = 0 \quad [n, \text{ any integer} \neq -1]$$

where C is the circle $|z - a| = r$. 7

**15.** Expand

$$f(z) = \frac{1}{(z-1)(z-2)}$$ 7

in the region

(a) $|z| < 1$, and

(b) $1 < |z| < 2$.