B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

00713

Term-End Examination

June, 2018

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of statistical tables and calculator is permitted.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate the limit

$$\lim_{x \to \infty} x \tan\left(\frac{1}{x}\right)$$

(b) If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, compute $\frac{dy}{dx}$.

(c) If $f(x, y, z) = 0$,

prove that $\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial x}{\partial y}\right)_z = -1$

(d) If $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$,

show that $\frac{\partial(y_1, y_2, y_3)}{\partial(x_1, x_2, x_3)} = 4$.

(e) Solve the differential equation

$(3x^2 + 2e^y)\, dx + (2xe^y + 3y^2)\, dy = 0$.

(f) Solve the differential equation

$\frac{dy}{dx} + y \tan x = x^2 e^x \cos x$.

2. Answer any ***four*** of the following : *4×4=16*

(a) Find the angle between the surfaces $x \log z = y^2 - 1$ and $x^2 y = 2 - z$ at the point (1, 1, 1).

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, show that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$.

(c) Show that the vector $\vec{v} = (y^2 - x^2 + y)\hat{i} + x(2y + 1)\hat{j}$ is irrotational.

(d) Evaluate the surface integral $\iint_S \vec{F} \cdot \hat{n}\, dA$ where $\vec{F} = z^2\hat{i} + xy\hat{j} - y^2\hat{k}$ and S is the portion of the surface of the cylinder $x^2 + y^2 = 36$, $0 \le z \le 4$ included in the first octant.

(e) Let D be the region bounded by the closed cylinder $x^2 + y^2 = 16$, $z = 0$ and $z = 4$. Verify the divergence theorem if

$$v = 3x^2 \hat{i} + 6y^2 \hat{j} + z\hat{k}.$$

(f) Evaluate the integral $\iint_S (\nabla \times \vec{v}).\hat{n}\, dA$ by Stoke's theorem

$\vec{v} = (x^2 - y^2)\hat{i} + (y^2 - x^2)\hat{j} + z\hat{k}$.

S is the portion of the surface $x^2 + y^2 - 2by + bz = 0$, where b is constant, whose boundary lies in the xy plane.

3. Answer any ***six*** of the following : *6×3=18*

(a) Find the inverse and adjoint of the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$

(b) Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 3 & 10 \end{bmatrix}.$$

(c) Find the non-singular matrices P and Q such that the normal form of A is PAQ, where

$$A = \begin{bmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{bmatrix}.$$

(d) Test if the system is consistent or inconsistent. If consistent then find the solution

$3x_1 + 2x_2 + x_3 = 3, \; 2x_1 + x_2 + x_3 = 0,$

$6x_1 + 2x_2 + 4x_3 = 6.$

(e) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}.$$

(f) Verify the Cayley-Hamilton theorem and find the inverse of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$$

(g) Show that

$$A = \begin{bmatrix} 2 & 3+4i \\ 3-4i & 2 \end{bmatrix}$$

is Hermitian.

(h) Solve by the Cramer's rule, $x + y + z = 6$, $2x - 3y + 4z = 8$, $x - y + 2z = 5$.

4. Answer any ***four*** of the following : *4×4=16*

(a) Out of 10 girls in a class, 3 have blue eyes. If 2 of the girls are chosen at random, what is the probability that (i) both have blue eyes ? (ii) neither has blue eyes ?

(b) If A and B be events with $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A \cup B) = \frac{1}{2}$, find (i) $P(A/B)$. (ii) $P(B/A)$.

(c) The probability that a pen manufactured by a company will be defective is 0·2. If 2 such pens are examined, find the probability that (i) exactly two, (ii) at least two, and (iii) none will be defective.

(d) A manufacturer of cotter pins knows that 5% of his product is defective. Pins are sold in boxes of 100. He guarantees that not more than 10 pins will be defective. Determine the probability that a box will fail to meet the guarantee.

(e) Can we conclude that the two population variances are equal for the following data of post graduates that passed out from a State and Private university :

State : 8350 8260 8130 8340 8070

Private: 7890 8140 7900 7950 7840 7920

(f) It has previously been recorded that the average depth of the ocean at a particular region is 67·4 fathoms. Is there reason to believe this at 0·01 level of significance, if the reading at 40 random locations in that particular region showed a mean of 69·3 with s.d. of 5·4 fathoms ?