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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

00592

June, 2018

ET-102 : MATHEMATICS – III

Time : 3 hours *Maximum Marks : 70*

Note : *Question no. 1 is **compulsory**. Attempt any other **eight** questions from question nos. 2 to 15. Use of calculator is allowed.*

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1. Fill in the blanks. ***All*** parts are to be attempted. *7×2=14*

(a) An infinite series $\sum_n x_n$ is said to be conditionally convergent if ________.

(b) A positive term series $\sum x_n$ be such that $\lim_{n \to \infty} (x_n)^{\frac{1}{n}} = l$, then the series diverges for ________.

(c) A function f(x) is defined in the interval $0 \leq x < \pi$ and if we take $f(-x) = -f(x)$ in the interval $0 \leq x < \pi$, we obtain ________ function for which Fourier coefficients ________ are zero.

(d) The differential equation, with solution $y = x^2 e^x + 4 \cos 2x$, is ________.

(e) The Laplace transform of function $(4 \cos^2 2t)$ is ________.

(f) Solution of differential equation

$$(D - 1)^2 (D + 1)^2 y = 0, \text{ with } D \equiv \frac{d}{dx},$$

is ________.

(g) The residue of the function $f(z) = \dfrac{z+2}{(z+1)^2 (z-2)}$ at the pole $z = -1$ is ________.

2. (a) Test the convergence of the series $3\frac{1}{2}$

$$\frac{\sqrt{n-1}}{\sqrt{n^3+1}} x^n, x > 0.$$

(b) Show that $\sum_n \dfrac{\cos nx}{n^2}$ is absolutely convergent and hence convergent. $3\frac{1}{2}$

3. Find the half-range cosine series for the function 7

$f(x) = (2x - 1)$, for $0 < x < 1$

4. Find the Fourier Series generated by the periodic function $|x|$ of period 2π. 7

5. (a) Determine the analytic function

$w = f(z) = u + iv$ if $v = -\dfrac{y}{x^2 + y^2}$. 4

(b) For the function $f(z) = \dfrac{2z^3 + 1}{z^2 + z}$, find Laurent's series expansion with the annulus $0 < |z| < 1$. 3

6. Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x \cos x - a \sin x}{x^2 + a^2} dx$, using complex variables and residue theorems. 7

7. (a) Evaluate $\displaystyle\int_{0}^{2\pi} \frac{(1 + 2\cos\theta)^n \cos n\theta}{3 + 2\cos\theta} d\theta$ 4

(b) Find the critical points and magnificient coefficient of conformal transformation $w = z^2 + 2z$ at the point $(1 - i)$. 3

8. Use the method of variation of parameters to solve the differential equation

$$(D^2 + a^2)\, y = \sec ax,\ D \equiv \frac{d}{dx}$$ 7

9. Find the series solution, near x = 0, of the differential equation.

$$x\,(1 - x)\, y'' + (1 - x)\, y' - y = 0$$ 7

10. Solve $(D^2 - 3DD' + 2D'^2)\, z =$

$e^{2x - y} + e^{x + y} + \cos(x + 2y)$, with $D \equiv \frac{\partial}{\partial x}$,

$D' \equiv \frac{\partial}{\partial y}$. 7

11. Using Laplace Transform, solve the differential equation

$y'' + 2y' + 5y = e^{-t} \sin t$, given $y(0) = 0$, $y'(0) = 1$. 7

12. Find the deflection of the vibrating string of length π, ends fixed corresponding to zero initial velocity and 3 sin 4x as initial deflection. 7

13. Evaluate :

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$$

by using convolution theorem. 7

14. For a series circuit with given values of inductance L, resistance R and elastance $\frac{1}{C}$ and an impressed voltage $E_0 \cos \omega t$, for what values of ω will the steady state current be a maximum ? 7

15. Find the characteristic function, transfer function, frequency response function and characteristic roots of the equation

$$(D + 4D^{-1})\, x = e^{3t}$$ 7
