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ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

00232

June, 2018

ET-101(A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are **compulsory**. Use of calculator is allowed.*

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1. Answer any ***five*** of the following : *5×4=20*

(a) Find

$$\lim_{x \to 0} \frac{x^2}{\sec x - 1}$$

(b) Find the value of 'b' for which the function

$$f(x) = \begin{cases} x^3 + 1, & \text{when } x < 2 \\ bx + \dfrac{2}{x}, & \text{when } x \geq 2 \end{cases}$$

is continuous at x = 2.

(c) Find the derivative of

$$\sin\left(\log \frac{2x}{x^2 - 3}\right).$$

(d) Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point (x_1, y_1).

(e) Find the shortest distance from the origin to the surface $xyz^2 = 2$.

(f) If

$$x = r \cos \theta,\ y = r \sin \theta, \text{ and } z = z,$$

find

$$\frac{\partial (x, y, z)}{\partial (r, \theta, z)}.$$

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate

$$\int e^{2x} . \sin 5x \, dx$$

(b) Evaluate

$$\int_0^{\pi/2} \sin^4 x \cos^5 x \, dx$$

(c) Find the area bounded by the parabola $y = x^2$ and the line $y = x$.

(d) From Simpson's rule with $h = 0 \cdot 1$, prove that $\int_0^1 \frac{dx}{1+x^2} = \frac{\pi}{4}$

(e) Solve the equation

$(1 - \sin x \tan y)\, dx + (\cos x \sec^2 y)\, dy = 0$

(f) Solve the differential equation

$$2\frac{dy}{dx} - \frac{y}{x} = 5x^3y^3.$$

3. Answer any ***four*** of the following : *4×4=16*

(a) Find the value of constant λ so that the vectors

$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$,

$\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$,

$\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar.

(b) Find the directional derivative of

$x^2 + y^2 + 4xy$ at $(1, -2, 2)$ in the direction of $2\hat{i} - 2\hat{j} + \hat{k}$.

(c) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that

(i) $\operatorname{div} \vec{r} = 3$,

(ii) $\operatorname{div}(\vec{r}/r^3) = 0$

(d) Show that the vector field defined by $\vec{F} = 2xyz^3 \hat{i} + x^2z^3 \hat{j} + 3x^2yz^2 \hat{k}$ is irrotational.

(e) Use Green's theorem to evaluate

$$\oint_C [(x^2 + xy)\, dx + (x^2 + y^2)\, dy]$$

where C is the boundary of the square $y = \pm 1, x = \pm 1$.

(f) Use divergence theorem to evaluate the surface integral $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ and S is $x^2 + y^2 + z^2 = a^2$ in first octant.

4. Answer any ***six*** of the following : *6×3=18*

(a) Let

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}.$$

Then show that both A and B are symmetric, but AB is not symmetric.

(b) Find the rank of A, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}.$$

(c) Find the inverse of matrix A by Gauss-Jordan method, where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}.$$

(d) Solve the system of linear equations by matrix method

$2x + y - z = 0,\ 2x + 5y + 7z = 52,\ x + y + z = 9.$

(e) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}.$$

(f) Verify Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

and hence compute A^{-1}.

(g) If

$$A = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$$

show that $(I - A)(I + A)^{-1}$ is a unitary matrix.

(h) Verify that the following matrix is orthogonal :

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$$