# MCA (Revised) <br> Term-End Examination 

## ロロ995 <br> June, 2018

## MCS-031 : DESIGN AND ANALYSIS OF ALGORITHMS

Time : 3 hours
Maximum Marks : 100
Note: Question no. 1 is compulsory. Attempt any three questions from the rest.

1. (a) Multiply the following $n$-digit decimal numbers, $x$ and $y$, using the Karatsuba technique, where

$$
\begin{align*}
& x=x_{n-1}, x_{n-2}, \ldots, x_{0} \\
& y=y_{n-1}, y_{n-2}, \ldots, y_{0} . \tag{7}
\end{align*}
$$

(b) Sort the following sequence in ascending order using Insertion sort :
$\{28,13,12,25,38,11,15,9,36\}$
(c) Differentiate between Asymptotic notations, O (Big "oh"), $\Omega$ (Big "omega") and $\Theta$ (Theta) notations.
(d) Define Minimum Cost Spanning Tree (MCST). Find the MCST using the Prim's algorithm for the following graph :

(e) Construct a Deterministic Finite Automata (DFA) for the following Regular Expression :

$$
(0+1)^{*}(00+11)(0+1)^{*}
$$

(f) Write the Recursive and Iterative algorithm to compute the Greatest Common Divisor (GCD) of two numbers X and Y .
(g) Explain the V.Strassen's matrix multiplication method of multiplying two matrices of size ( $n \times n$ ). Show that its running time is $\mathrm{O}\left(\mathrm{n} \log _{2} 7\right)$.
2. (a) Write the Dijkstra's algorithm for shortest path. Apply the same for the following graph, to find the shortest path from node (a):

(b) Explain the Chomsky's classification of grammars.
(c) Differentiate between the Greedy and Dynamic programming approaches to solve a problem. List 3 problems which use these approaches, respectively.
3. (a) Write the Context-Free Grammar (CFG) for the following language :
(i) $L=\left\{a^{n} b^{m} c^{m} d^{n} \mid m, n \geq 1\right\}$
(ii) $\mathrm{L}=\left\{\omega \mathrm{d} \omega^{\mathrm{R}} \mid \omega \in\{\mathrm{a}, \mathrm{b}\}^{*}\right.$ and $\omega^{\mathrm{R}}$ is the reverse of $\omega$ \}.
(b) Explain the 0/1 Knapsack problem. Solve the following $0 / 1$ Knapsack problem :
Given number of objects $n=6$
Capacity of Knapsack (M) $=12$
$\left(p_{1}, p_{2}, \ldots, p_{6}\right)=(1,6,18,22,28,43)$
and $\left(w_{1}, w_{2}, \ldots, w_{6}\right)=(1,2,5,6,7,10)$.
Where $\mathrm{p}_{i}$ 's and $\mathrm{w}_{i}$ 's are the profits and weights of the corresponding objects.
(c) Apply the dynamic programming
method to solve the following chain-matrix-multiplication :
$\left(M_{1}, M_{2}, M_{3}, M_{4}\right)$ with dimensions
$(15 \times 6,6 \times 50,50 \times 9,9 \times 12)$
4. (a) Solve the following recurrence using the Master method :
(i) $T(n)=3 T\left(\frac{n}{4}\right)+n \log n$
(ii) $T(n)=2 T\left(\frac{n}{2}\right)+n$
(b) Find the best case and worst case time complexity of Quick Sort by writing their recurrence relation.
(c) Define Turing Machine (TM). Design. TM for the language

$$
\begin{equation*}
L=\left\{a^{n} b^{n} \mid n \geq 1\right\} \tag{7}
\end{equation*}
$$

5. (a) Differentiate between P, NP, NP-Complete and NP-Hard problem.
(b) Define Push-Down Automata (PDA).

Design a PDA that accepts the language EVEN PALINDROME over $\Sigma=\{\mathrm{a}, \mathrm{b}\}$.
(c) Write short notes on the following :
(i) Halting Problem of TM
(ii) Post Correspondence Problem (PCP)

