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BCS-012

BACHELOR OF COMPUTER APPLICATIONS (BCA) (Revised)

Term-End Examination

June, 2018

02735

BCS-012 : BASIC MATHEMATICS

Time : 3 hours *Maximum Marks : 100*

Note : *Question number 1 is* ***compulsory****. Attempt any* ***three*** *questions from the rest.*

1. (a) Show that

$$\begin{vmatrix} 1 & a & a^2 \\ a^2 & 1 & a \\ a & a^2 & 1 \end{vmatrix} = (a^3 - 1)^2$$ 5

(b) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. 5

(c) Find the sum up to n terms of the series

$3 + 33 + 333 + \ldots$ 5

(d) If 1, ω, ω^2 are the cube roots of unity, show that $(1 + \omega + \omega^2)^5 + (1 - \omega + \omega^2)^5 + (1 + \omega - \omega^2)^5 = 32$ 5

(e) If $y = 1 + \ln (x + \sqrt{x^2 + 1})$, prove that

$$(x^2 + 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0.$$ 5

(f) A stone is thrown into a lake producing a circular ripple. The radius of the ripple is increasing at the rate of 5 m/s. How fast is the area inside the ripple increasing when the radius is 10 m ? 5

(g) Find the value of λ for which the vectors $\vec{a} = \hat{i} - 4\hat{j} + \hat{k}$, $\vec{b} = \lambda\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + 3\hat{k}$ are coplanar. 5

(h) Find the angle between the lines

$$\vec{r} = 2\hat{i} + 3\hat{j} - 4\hat{k} + t(\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\vec{r} = 3\hat{i} - 5\hat{k} + s(3\hat{i} - 2\hat{j} + 6\hat{k}).$$ 5

2. (a) Solve the following system of equations by the matrix method :

$2x - y + 3z = 5$, $3x + 2y - z = 7$,

$4x + 5y - 5z = 9$. 5

(b) Show that $A = \begin{bmatrix} 3 & 4 & -5 \\ 3 & 3 & 0 \\ 1 & 1 & 5 \end{bmatrix}$ is row equivalent to I_3. 5

(c) Use the principle of mathematical induction to show that

$$1 + 4 + 7 + \ldots + (3n - 2) = \frac{1}{2} n (3n - 1).$$ 5

(d) Find the quadratic equations with real coefficients and with the following pair of roots : $\frac{m - n}{m + n}, -\frac{m + n}{m - n}$ 5

3. (a) Evaluate : 5

$$\lim_{x \to 0} \frac{\sqrt{1 + 2x} - \sqrt{1 - 2x}}{x}$$

(b) If $(x + iy)^{1/3} = a + ib$, prove that

$$\frac{x}{a} + \frac{y}{b} = 4 (a^2 - b^2)$$ 5

(c) Solve the equation

$$2x^3 - 15x^4 + 37x - 30 = 0,$$

if the roots of the equation are in A.P. 5

(d) Draw the graph of the solution set of the following inequalities : 5

$$2x + y \geq 8, \; x + 2y \geq 8 \text{ and } x + y \leq 6.$$

4. (a) Determine the values of x for which the following function is increasing and for which it is decreasing :

$$f(x) = (x - 1) (x - 2)^2$$ 5

(b) Find the absolute maximum and minimum of the following function :

$$f(x) = \frac{x^3}{x+2} \text{ on } [-1, 1].$$ 5

(c) Find the length of the curve $y = 2x^{3/2}$ from the point (1, 2) to (4, 16). 5

(d) Evaluate the integral

$$\int \frac{(x+1)^2}{(x-1)^2} dx$$ 5

5. (a) If $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - \hat{k}$, verify that

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}.$$ 5

(b) Find the vector and Cartesian equations of the line passing through the points (– 2, 0, 3) and (3, 5, – 2). 5

(c) Reduce the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

to its normal form and hence determine its rank. 5

(d) Find the direction cosines of the line passing through the two points (1, 2, 3) and (– 1, 1, 0). 5