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MST-003

POST GRADUATE DIPLOMA IN APPLIED STATISTICS (PGDAST)

Term-End Examination

02552

June, 2017

MST-003 : PROBABILITY THEORY

Time : 3 hours *Maximum Marks : 50*

Note :

*(i) Attempt **all** questions. Questions no. 2 to 5 have internal choices.*

(ii) Use of scientific calculator is allowed.

(iii) Use of Formulae and Statistical Tables Booklet for PGDAST is allowed.

(iv) Symbols have their usual meanings.

1. State whether the following statements are *True* or *False*. Give reasons in support of your answers.

5×2=10

(a) For any two events A and B, the probability $P(A|B) + P(\overline{A}|B) = 1/4$.

(b) For a Poisson variate X with parameter 2, the standard deviation and mean are 2 and $\sqrt{2}$.

(c) If X ~ N(5, 9) and Y ~ N(3, 16) are two independent normal variates, then (X – Y) ~ N(8, 7).

(d) If the probability density function of a variable x is f(x) = Cx (2 – x), where $0 \leq x \leq 2$, then the value of C is found to be 3/4.

(e) If the probability that a student will get grade 'A' in a statistics course is 0·32, then the probability that he will get either grade A or grade B may be 0·27.

2. (a) If there are 5 people in a room, what is the probability that no two of them have the same date of birth ? *3*

(b) Three dice are rolled. If no two dice show the same face, what is the probability that one dice shows face one ? *3*

(c) A die is of the shape of a regular tetrahedron whose faces have the numbers 111, 112, 121, 122. A_1, A_2, A_3 are respectively, events that the first two, the last two and the extreme two digits are the same, when the die is tossed at random. Find whether or not the events A_1, A_2, A_3 are pairwise independent. *4*

OR

(a) By considering a suitable example, describe the application of the law of total probability. *3*

(b) In a bolt factory, machines A, B and C manufacture respectively, 25, 35 and 40 percent of the total product. Of their output 5, 4 and 2 percent respectively are defective bolts. A bolt is randomly selected from a lot and is found to be defective. What is the probability that the bolt selected was manufactured by (i) machine A, (ii) machine B, and (iii) machine C ? *7*

3. (a) A discrete random variable X is defined as follows :

X :	0	1	2	3	4
P(X = x) :	K	3K	0·2	K	2K + 0·1

Find (i) the value of K and the probability distribution of X, and (ii) $P(X > 2)$. *4*

(b) The distribution function of a variable X is as follows :

$$F(x) = 0 \quad \text{if } x < 0$$
$$= \frac{x}{8} \quad \text{if } 0 \le x < 2$$
$$= \frac{x^2}{16} \quad \text{if } 2 \le x < 4$$
$$= 1 \quad \text{if } x \ge 4$$

Find E(X) and Variance (X). *6*

OR

(a) If $f(x) = 2x$ when $0 \leq x \leq 1$

$= 0$ otherwise,

find

(i) $P(X < \frac{1}{2})$,

(ii) $P(\frac{1}{4} < X < \frac{1}{2})$,

(iii) $P(X > \frac{3}{4}$ given $X > \frac{1}{2})$. *3*

(b) Let X be a random variable with pdf

$$f(x) = C\,(1 - x^2) \text{ where } 0 < x < 1.$$

Then find (i) C, (ii) E(X), (iii) $P[\frac{1}{2} < X < \frac{3}{4}]$. *5*

(c) Define random variable. Describe the types of random variables with suitable examples. *2*

4. (a) An irregular 6-faced die is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets ? *5*

(b) If the probability that an applicant for a driver's licence will pass the road test on any given trial is 0·8, what is the probability that he will finally pass the test (i) on the fourth trial, and (ii) in fewer than 4 trials ? *5*

OR

(a) It is known that the probability of an item produced by a certain machine will be defective is 0·05. If the produced items are sent to the market in packets of 20, find the number of packets containing (i) at least, (ii) exactly, and (iii) at most 2 defective items in a consignment of 1000 packets using Poisson distribution. *6*

(b) A panel of 7 judges is to decide which of the 2 final contestants A and B will be declared the winner. A simple majority of the judges will determine the winner. Assume that 4 of the judges will vote for A and the other 3 will vote for B. If we randomly select 3 of the judges and seek their verdict, what is the probability that a majority of them will favour A ? *4*

5. (a) Buses arrive at a specified stop at 15 minute intervals starting at 7 a.m., that is, they arrive at 7:00, 7:15, 7:30, 7:45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7:00 and 7:30 a.m., find the probability that he waits (i) less than 5 minutes for a bus, and (ii) at least 12 minutes for a bus. *6*

(b) If the actual amount of instant coffee which a filling machine puts into '6-ounce' jars is a random variable having a normal distribution with standard deviation 0·05 ounce and if only 3% of the jars are to contain less than 6 ounces of coffee, what must be the mean fill of these jars ? *4*

OR

(a) The percentage X of a particular compound contained in a rocket fuel follows the distribution N(33, 3), though the specification for X is that it should lie between 30 and 35. The manufacturer will get a net profit (per unit of the fuel) of ₹ 100, if $30 < X < 35$, of ₹ 50, if $25 < X \leq 30$ or $35 \leq X < 40$ and incur a loss of ₹ 60 per unit of fuel otherwise. Find the expected profit of the manufacturer. *7*

(b) The time (in hours) required to repair a machine is exponentially distributed with $\lambda = 1/2$.

(i) What is the probability that the repair time exceeds 2 hours ?

(ii) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours ? *3*
