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MMT-009

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)

00922

M.Sc. (MACS)

Term-End Examination

June, 2017

MMT-009 : MATHEMATICAL MODELLING

Time : $1\frac{1}{2}$ hours *Maximum Marks : 25*

(Weightage : 70%)

Note : *Answer any **five** questions. Use of calculator is **not** allowed.*

1. (a) Find a linear demand equation that best fits the following data and use it to predict annual sales of cars at ₹ 12,00,000 : 3

x = Price (Lacs of rupees)	3	5	8	15	20	32
y = Sales of new cars in this year	158	94	75	60	80	16

(b) Find the number of quantities required for estimating the expected return and standard deviation for 300 securities in Markowitz model. How many estimates are required for these securities while using single index Sharpe model ? *2*

2. (a) The reproduction function of cancer cells within a spherical tumour is given by

$$\phi(c) = \frac{2c + 1}{(1 - 3c)^2}, \ c \neq \frac{1}{3},$$

with initial conditions given by $c = c_0$ at $t = 0$. Find the density of cancer cells in the tumour's surface area at $t = 45$ days. *3*

(b) Given below are different problems related to tumour growth. Identify the type of model (deterministic, continuous, stochastic or discrete) which is most appropriate in each of the following situations. Give reasons for your answer. *2*

(i) Effect of the treatment given at regular intervals.

(ii) Effect of drugs on a patient who is given the drug for a given duration of time.

(iii) Effect of radiation on tumour cells; some cells may continue to grow but some may be damaged.

(iv) Finding the time taken for a tumour to double in size.

3. A bank has 4 counters to receive people, who come to deposit and withdraw money, have fixed deposit accounts and deposit government taxes. On an average 72 persons arrive in a 6-hour day. Each counter officer spends 15 minutes on an average on an arrival. If the arrivals are Poisson distributed and service times are according to exponential distribution, find

(a) the average number of customers in the system,

(b) the average number of customers to be served,

(c) the average working time for a customer,

(d) the number of hours each week that a counter officer spends performing his job, and

(e) the expected number of idle counter officers at any specified time. 5

4. Consider the delay model of a population growth given by the difference equation

$$U_{n+1} = U_n \exp\left[r\left(2 - \frac{U_{n-1}}{2}\right)\right], r > 0.$$

Find the linear steady-states of the model and do the stability analysis when $0 < r < \frac{1}{8}$. 5

5. Do the stability analysis of the following interacting system of species under the effect of toxicant, when the concentration of the toxicant in the environment is assumed to be constant :

$$\frac{dN_1}{dt} = r_1N_1 - \alpha_1N_1N_2 - d_1C_0N_1$$

$$\frac{dN_2}{dt} = r_2N_2 - \alpha_2N_1N_2 - d_2V_0N_2$$

$$\frac{dC_0}{dt} = k_1P - g_1C_0 - m_1C_0$$

$$\frac{dV_0}{dt} = k_2P - g_2V_0 - m_2V_0$$

under the initial conditions

$N_1(0) = N_{10}, N_2(0) = N_{20}, C_0(0) = 0, V_0(0) = 0.$

The variables and parameters notation in the above system of equations are

$N_1(t), N_2(t)$ = Densities of two different populations

$C_0(t)$ = Concentration of the toxicant in the individuals of the population $N_1(t)$

$V_0(t)$ = Concentration of the toxicant in the individuals of the population $N_2(t)$

P = Concentration of the toxicant in the environment and is constant

r_1 and r_2 are the birth rates; α_1, α_2 are the predation rates; d_1 is the death rate due to C_0; d_2 is the death rate due to V_0; k_1, k_2 are the uptake rates; g_1, g_2 are the loss rates; m_1, m_2 are the depuration rates.

Here r_1, r_2, α_1, α_2, d_1, d_2, k_1, k_2, g_1, g_2, m_1, m_2, and P are all positive constants. *5*

6. (a) Let σ_1 and σ_2 be the standard deviations of two securities 1 and 2, respectively. If $\sigma_1 = \sigma_2$ and the standard deviation σ_P of the portfolio $P = (w_1, w_2)$ is minimum, then show that $w_1 = w_2 = 0{\cdot}5$, w_1 and w_2 are the portfolio weights of securities 1 and 2, respectively. $2\frac{1}{2}$

(b) In a population of birds, the proportionate birth rate and death rate are both constants being 0·45 and 0·65 per year, respectively. Immigration occurs at a constant rate of 2000 birds and emigration at a constant rate of 1000 birds per year. Use these assumptions to formulate a model of the population. Solve the model and describe the long-term behaviour of the population. $2\frac{1}{2}$