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MMT-008

M.Sc. (MATHEMATICS WITH APPLICATIONS IN COMPUTER SCIENCE)
M.Sc. (MACS)

00682

Term-End Examination

June, 2017

MMT-008 : PROBABILITY AND STATISTICS

Time : 3 hours *Maximum Marks : 100*

(Weightage : 50%)

Note : *Question no. 8 is* ***compulsory****. Answer any* ***six*** *questions from questions no. 1 to 7. Use of calculator is* ***not*** *allowed. All the symbols used have their usual meaning.*

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1. (a) A cashier in a hospital handles all the payments of the patients. Customers arrive to the cashier at an average of 18 per hour. The service time per customer is, on an average, 2 minutes. Find the following : 5

   (i) Average size of the queue at the cashier window excluding the person getting service.

(ii) Probability that the cashier is idle.

(iii) Average time spent by a new arrival for payment.

(iv) Probability that a customer finds 3 persons ahead of him/her including the one getting service when he arrives.

(b) In a die-coin experiment, a fair die is rolled and then a fair coin is tossed a number of times equal to the score of a die. Find the probability that the coin shows tail at every toss. *6*

(c) A particle moves on a circle through points which have been marked 0, 1, 2, 3, 4 (in a clockwise order). At each step it has a probability p of moving to the right (clockwise) and $1 - p$ to the left (anticlockwise). Let X_n denote its location on the circle after the n^{th} step. The process $\{X_n, n \geq 0\}$ is a Markov chain.

(i) Find the transition probability matrix.

(ii) Calculate the limiting probabilities. *4*

2. (a) The joint probability mass function of random variables X and Y is given in the following table :

y \ x	0	1	2
– 2	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{12}$
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$
2	$\frac{1}{6}$	0	$\frac{1}{12}$

(i) Find the marginal distributions of X and Y.

(ii) Find E(X) and E(Y).

(iii) Test the independence of X and Y.

(iv) Find Cov (X, Y). *8*

(b) A bivariate population has the following mean vector and variance-covariance matrix :

$$\mu = \begin{bmatrix} 40 \\ 10 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 13 & 10 \\ 10 & 6 \end{bmatrix}$$

A sample of 10 observations from the population gives sample mean

$$\overline{X} = \begin{bmatrix} 33 \\ 7 \end{bmatrix}.$$

Test whether the sample confirms its truthfulness of mean vector at 5% level of significance. [You may like to use the values, $\chi^2_{2, 0\cdot05} = 10\cdot6$, $\chi^2_{1, 0\cdot05} = 7\cdot88$] *7*

3\. (a) π_1 and π_2 are two populations with probability density functions $p_1(x)$ and $p_2(x)$, respectively. The cost of misclassification of an item in population π_2, given that it was from π_1 is ₹ 100 and the cost of misclassification in population π_1, given that it was from π_2 is ₹ 30. It is known that 70% of the items belong to population π_1.

(i) Find the prior probabilities.

(ii) Find the classification regions.

(iii) If an item has $p_1(x) = 0 \cdot 2$ and $p_2(x) = 0 \cdot 9$, then classify it in π_1 or π_2. 7

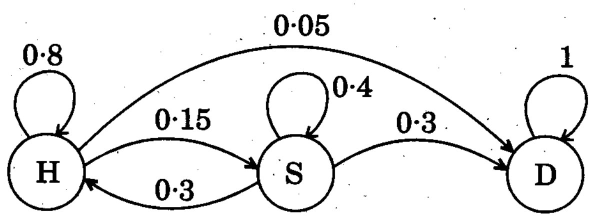
(b) Let $X \sim N_3(\mu, \Sigma)$ where $\mu' = [3, 1, 5]$ and

$$\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

(i) Are X_1 and X_3 independent ? Why ?

(ii) Obtain the distribution of CX where $C = \begin{bmatrix} -1 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$. 8

4. (a) The transition graph of a Markov chain having three states; healthy, sick and dead is given below. The graph given below provides the transition probabilities for the changes in the condition of patients in a hospital in a week.



If the probability that a patient is healthy be 0·7, then find

(i) the probability that the patient is found healthy and will be sick in the coming week.

(ii) the probability that a patient is found healthy and will be sick in the next two weeks.

(iii) Write the transition probability matrix. *7*

(b) Let $\{X_n, n = 1, 2, ... \}$ be i.i.d. geometric variables with probability mass function of each as $p_i = (1 - p)\, p^{i-1}, i = 1, 2, ...$.

(i) Find the number of renewals which follow binomial distribution in the corresponding renewal process.

(ii) Find the renewal function of the corresponding renewal process. *4*

(c) If X and Y are two random variables with $E[Y|X] = 1$, show that $Var(XY) \geq Var(X)$. 4

5. (a) Find the principal components and proportions of total population variance explained by each component when the covariance matrix is given by $\Sigma = \begin{bmatrix} 3 & 2 \\ 2 & 6 \end{bmatrix}$. 7

(b) At 5% level of significance, test $\mu = [7, 11]$ using T^2 from the mean $\overline{X}$ and S^{-1} of a sample of size 10, which are given below :

$$\overline{X} = \begin{pmatrix} 6 \\ 12 \end{pmatrix} \quad S^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{8} \end{bmatrix}.$$

[You may like to use the values

$F_{2, 8, 0\cdot05} = 4\cdot46$, $F_{1, 8, 0\cdot05} = 5\cdot82$] 8

6. (a) In a branching process, offspring distribution follows a geometric distribution as given below :

$$p_k = pq^k;\ q = 1 - p,\ k = 0, 1, 2, ...$$

Find the probability of extinction given that

(i) $p = 0\cdot2$

(ii) $p = 0\cdot7$ 7

(b) The transition probability matrix of a Markov chain having three states 1, 2, 3 is given below :

$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & 0 \end{bmatrix}$$

(i) Are all states communicable ? Why ?

(ii) Determine the closed set.

(iii) Is the chain irreducible ? Why ?

(iv) Find the probability of ultimate return to the state 1.

(v) Find the mean recurrence time of the state. *8*

7. (a) A drug test is 99% sensitive, that is, positive result is true for drug users and 99% specific, that is, negative result is true for non-drug users. Assume that 1% are drug users. If a randomly selected person is found positive to the test, what is the probability of his/her being a drug user ? *4*

(b) Let $X' = (X_1, X_2, X_3)$ be a random vector and X be the data matrix given below :

$$X = \begin{bmatrix} 5 & 3 & 4 \\ 2 & 1 & 3 \\ 5 & 6 & 4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

Find

(i) the variance-covariance matrix Σ,

(ii) the correlation matrix R. *8*

(c) Consider the mean vector $\mu_x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $\mu_y = 2$. The covariance matrices of x_1, x_2 and y are

$$\Sigma_{xx} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix},\ \sigma_{yy} = 9,\ \sigma_{xy} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

Fit the equation $y = b_0 + b_1x_1 + b_2x_2$ as best linear equation. *3*

8. State whether the following statements are *True* or *False*. Justify your answers. *5×2=10*

(a) The variance-covariance matrix of a random vector of dimension 2 is given by

$$\begin{bmatrix} 1 & -2 \\ -2 & 2 \end{bmatrix}.$$

(b) If the probability of critical region under null hypothesis H_0 is α, then the probability under alternate hypothesis H_1 is $1 - \alpha$.

(c) A sequence of independent random variables does not form a Markov chain.

(d) The matrix of the quadratic form

$$x_1^2 + 2x_1x_2 + x_2^2 \text{ is } \begin{bmatrix} 1 & \sqrt{2} \\ \sqrt{2} & 1 \end{bmatrix}.$$

(e) For any two events A and B, $P(A) > P(A | B)$.