

**M.Sc. (MATHEMATICS WITH APPLICATIONS
IN COMPUTER SCIENCE)
M.Sc. (MACS)**

00132 **Term-End Examination**
June, 2017

MMT-005 : COMPLEX ANALYSIS

Time : $1\frac{1}{2}$ hours

Maximum Marks : 25

Note : Question no. 1 is compulsory. Attempt any three questions from questions no. 2 to 5. Use of calculator is not allowed.

1. State, giving reasons, whether the following statements are *True* or *False* : $5 \times 2 = 10$

(a) The function $f(z) = \frac{1}{z}$ ($0 \neq z \in \mathbb{C}$) is differentiable everywhere in $\mathbb{C} - \{0\}$.

(b)
$$\int_{|z-i|=3} \frac{1}{(z-2)^3} dz \neq 0$$

- (c) If $f(z) = u + iv$ is analytic in a domain D and $u^4 + v^4 = \text{constant}$, then f is constant.
- (d) The harmonic conjugates of a function $u(x, y)$ differ by a constant.
- (e) $z = 0$ is a removable singularity of the function $\frac{\sin z}{z^m}$, where m is an integer, $m > 2$.

2. (a) Using the definition of contour integral, evaluate $\int_C z^2 dz$ where C is the join of the

straight line $[-2, 2]$ and the upper semicircle with centre 0 and radius 2 .

3

- (b) Show that $Tz = \frac{az + b}{cz + d}$, $ad - bc \neq 0$ has 0 and ∞ as its only fixed points iff it is a dilation.

2

3. (a) Evaluate $\int_C \frac{z}{z^2 + 9} dz$ where C is the circle

$$|z - 2i| = 4.$$

3

- (b) Expand $f(z) = \frac{1}{(z-1)^2(z-3)}$ in a Laurent series valid for $0 < |z-1| < 2$.

2

4. (a) Show that the mapping $w = \frac{1}{z}$ maps a circle not passing through the origin in the z -plane into a circle not passing through the origin in the w -plane. 3
- (b) Consider $f(z) = z(z - 3)$ and the closed circular region $R = \{z : |z| \leq 3\}$. Find the points in R where $|f(z)|$ has its maximum. 2
5. Evaluate : 5

$$\int_0^{\infty} \frac{x \sin x}{x^2 + 9} dx$$
