No. of Printed Pages : 5 **BME-001**

B.Tech. MECHANICAL ENGINEERING (COMPUTER INTEGRATED MANUFACTURING)

00834

Term-End Examination

June, 2017

BME-001 : ENGINEERING MATHEMATICS-I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of scientific calculator is allowed. Statistical tables are allowed.*

1. Attempt any *five* of the following : *5×4=20*

   (a) Evaluate the following limits :

   $$\lim_{x \to -3} \frac{x^3 - 27}{x + 3}$$

   (b) Prove that f(x) = sin x is continuous at x = 0.

   (c) Find the derivative of $y = 5x^2 \sin 2x$.

   (d) Use Maclaurin's formula and expand the function $y = \log_e (1 + x)$.

(e) If $u = x + y + z$, $y + z = uv$, $z = uvw$, find $\frac{\partial(x, y, z)}{\partial(u, v, w)}$ and $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

(f) Evaluate $\int_0^1 \frac{\tan^{-1} x}{1 + x^2} dx$.

(g) Solve the equation
$(1 - \sin x \tan y)\, dx + (\cos x \cdot \sec^2 y)\, dy = 0$.

2. Attempt any ***four*** of the following : *4×5=20*

(a) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point (1, – 1, 2).

(b) Find the directional derivative of $f = xy^2 + yz^3$ at the point (2, – 1, 1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$.

(c) Prove that $\nabla^2 r^n = n(n + 1) r^{n-2}$, where n is a constant and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Hence prove that $\nabla^2 \frac{1}{r} = 0$.

(d) Evaluate $\int_C (y^2\, dx - 2x^2\, dy)$ along the parabola $y = x^2$ from (0, 0) to (2, 4).

(e) Evaluate $\int_C [(x + y)\, dx - x^2\, dy + (y + z)\, dz]$ where C is $x^2 = 4y$, $z = x$, $0 \le x \le 2$.

(f) Use the divergence theorem to evaluate

$$\iint_S (\vec{v} \cdot \vec{n})\, dA \text{ where}$$

$\vec{v} = x^2 z \hat{i} + y \hat{j} - xz^2 \hat{k}$ and S is the boundary of the region bounded by the parabola $z = x^2 + y^2$ and the plane $z = 4y$.

3. Answer any ***five*** of the following : *5×3=15*

(a) Given the matrices

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -5 & 2 \\ 1 & 7 & 1 \end{bmatrix}$$

verify that $|AB| = |A|\,|B|$.

(b) Find the inverse of

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}.$$

(c) Show that the system of equations

$$\begin{bmatrix} 1 & -1 & 3 \\ 2 & 3 & 1 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$$

has infinite number of solutions. Hence find the solution.

(d) Determine the rank of the matrix

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 8 & 4 & 6 \\ -2 & -1 & -1\cdot5 \end{bmatrix}.$$

(e) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}.$$

(f) Establish the following identity using the properties of the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c).$$

(g) Solve the system by Cramer's rule :

$x - y + z = 2,\ 2x + 3y - z = 5,\ x + y - z = 0$

4. Attempt any ***three*** of the following : *3×5=15*

(a) A, B and C can hit a target with probabilities $\frac{3}{5}$, $\frac{2}{5}$ and $\frac{3}{4}$, respectively. Determine the probability that (i) two shots hit, and (ii) at least two shots hit.

(b) In a bolt factory, machines A, B and C manufacture 25%, 35% and 40% of the total output respectively. Of their output 5%, 4% and 2% are defective bolts. A bolt is chosen at random and found to be defective. What will be the probability that the bolt came from machines A, B and C ?

(c) Ten percent of screws produced in a certain factory turn out to be defective. Find the probability that in a sample of 10 screws chosen at random, exactly two will be defective.

(d) A random sample of 400 tins of vegetable oil and labelled "5 kg net weight" has a mean net weight of 4·98 kg with standard deviation of 0·22 kg. Do we reject the hypothesis of net weight of 5 kg per tin on the basis of this sample at 1% level of significance ?

(e) Ten individuals are chosen at random from the population and their heights are found to be 63, 63, 64, 65, 66, 69, 69, 70, 70, 71 inches. Discuss the suggestion that the mean height in the universe is 65 inches given that for 9 degrees of freedom the value of student's 't' at 0·05 level of significance is 2·262.