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ET-102

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering)

Term-End Examination

June, 2017

00505

ET-102 : MATHEMATICS – III

Time : 3 hours *Maximum Marks : 70*

Note : *Attempt any **ten** questions. All questions carry equal marks. Use of calculator is allowed.*

1. Test the convergence of the series

$$\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \ldots$$

for all values of x. 7

2. Show that the series

$$\sum_n (-1)^n \sin\left(\frac{1}{n}\right)$$ is not absolutely convergent. 7

3. Find the half-range cosine series for the function

$f(x) = (2x - 1)$ for $0 < x < 1$. 7

4. Determine the analytic function $w = u + iv$ if $u - v = (x - y)(x^2 + 4xy + y^2)$ and express w in terms of z. 7

5. For the function

$$f(z) = \frac{2z^3 + 1}{z^2 + z},$$

find

(a) Taylor series expansion valid in the neighbourhood of $z = i$.

(b) Laurent's series expansion within the annulus when the centre is the origin. *3+4*

6. Using Cauchy's residue theorem, evaluate

$$\int_C \frac{z+2}{(z+1)^2 (z-2)} dz,$$

C being the circle $|z| = 3$. *7*

7. Evaluate

$$\int_C \frac{1 - e^{2miz}}{z^2(a^2 + z^2)^2} dz$$

to show that

$$\int_0^\infty \frac{\sin^2 mx}{x^2(a^2 + x^2)^2} dx = \frac{\pi}{8a^6}(e^{-2ma}(2ma + 3) + 4ma - 3),$$

where C consists of a semicircle in the upper half-plane together with the real axis indented at the origin. *7*

8. Find the bilinear transformation that maps i, 1, – 1 into 1, 0, ∞. 7

9. Find the Laplace Transform of the function f(t) defined as

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < 2 \end{cases}$$

with f(t + 2) = f(t) outside the given interval. 7

10. Show that

$$\mathcal{L}^{-1}\left[\frac{1}{s(s^2 + k^2)}\right] = \frac{1 - \cos kt}{k^2}.$$ 7

11. Find the characteristic function, transfer function, frequency response function and characteristic roots of the equation

$$(D - 2 + D^{-1})\, z = f.$$

Also test the above equation for stability. 7

12. Use Hurwitz-Routh criterion to determine the value of 'a', so that the differential equation whose characteristic function is given by

$$s^4 + as^3 + 3s^2 + 4s + 1 = 0$$

is stable. 7

13. Solve

$$xzp + yzq = xy,$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. 7

14. Solve

$$(D^2 + 3D - 4)\, y = xe^{-2x};\ D \equiv \frac{d}{dx}.$$ 7

15. Solve

$$\frac{dx}{dy} + \frac{x}{y \log y} = \frac{1}{y}.$$ 7

16. Solve

$$(x + 2y - 1)\, dx = (x + 2y + 1)\, dy.$$ 7
