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ET-101(A)

B.Tech. Civil (Construction Management) / B.Tech. Civil (Water Resources Engineering) / B.Tech. (Aerospace Engineering) / BTCLEVI / BTMEVI / BTELVI / BTECVI / BTCSVI

Term-End Examination

00405

June, 2017

ET-101(A) : MATHEMATICS – I

Time : 3 hours *Maximum Marks : 70*

Note : *All questions are* ***compulsory****. Use of scientific calculator is permitted.*

1. Answer any ***five*** of the following : *5×4=20*

(a) Evaluate any ***one*** of the following :

(i) $\lim_{x \to \infty} \left(1 + \frac{1}{x^2}\right)^x$

(ii) $\lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x}$

(b) For what value of b is

$$g(x) = \begin{cases} x & x < -2 \\ bx^2 & x \geq -2 \end{cases}$$

continuous at every x ?

(c) If $y = e^{a \sin^{-1} x}$,

prove that :

(i) $(1 - x^2) y_2 - xy_1 - a^2 y = 0$

(ii) $(1 - x^2) y_{n+2} - (2n + 1) xy_{n+1} - (n^2 + a^2) y_n = 0$

(d) If $y = \frac{x}{\log_e x}$, show that $x = e$ gives the minimum value of y.

(e) If $x^y = e^{x-y}$,

prove that

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}.$$

(f) If $x = r \cos \theta$, $y = r \sin \theta$,

evaluate

$$\frac{\partial (x, y)}{\partial (r, \theta)}.$$

(g) Show that the curves

$x^3 - 3xy^2 = 2$ and $3x^2y - y^3 = 2$ cut orthogonally.

(h) Find the volume of the largest possible right circular cylinder that can be inscribed in a sphere of radius R.

2. Answer any ***four*** of the following : *4×4=16*

(a) Evaluate any ***one*** of the following :

(i) $\int \frac{1}{(x-3)\sqrt{x+1}}\,dx$

(ii) $\int \sin^5 x\,dx$

(b) Evaluate any ***one*** of the following :

(i) $\int_1^2 \frac{x}{x^2+1}\,dx$

(ii) $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

(c) Find the value of

$$\int_2^{10} \frac{1}{1+x}\,dx,$$

by using Simpson's $\frac{1}{3}^{rd}$ rule using nine ordinates.

(d) Find the area bounded by the curves $y^2 = 9x$ and $x^2 = 9y$.

(e) Find the length of the curve
$x = a\cos^3\theta$, $y = a\sin^3\theta$,
in the first quadrant.

(f) Solve (any ***one***) :

(i) $x\frac{dy}{dx} + \cot y = 0$ given $y = \frac{\pi}{4}$, where $x = \sqrt{2}$

(ii) $\frac{dy}{dx} = (4x + y + 1)^2$

3. Answer any ***four*** of the following : *4×4=16*

(a) A particle moves along the curve $x = t^3 + 1$, $y = t^2$, $z = 2t + 3$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\hat{i} + \hat{j} + 3\hat{k}$.

(b) Find grad f at the point (1, 2, –1), where $f = x^2 y e^z$.

(c) Prove that $\vec{a} \cdot \nabla \mathbf{r}$ = constant, where $\vec{a}$ is a constant vector.

(d) If the vector

$$\mathbf{P} = (ax + 3y + 4z)\,\hat{i} + (x - 2y + 3z)\,\hat{j} + (3x + 2y - z)\,\hat{k}$$

is solenoidal, then determine the constant a.

(e) Show that $f = ax^2 + by^2 + cz^2$, satisfies Laplace's equation $\nabla^2 f = 0$, provided $a + b + c = 0$.

(f) If $\mathbf{F} = (x + 2y + az)\,\hat{i} + (bx - 3y - z)\,\hat{j} + (4x + cy + 2z)\,\hat{k}$, find a, b and c, so that $\mathbf{F}$ is irrotational. Also find its scalar potential.

4. Answer any ***six*** of the following : *6×3=18*

(a) Solve the equation for x, y, z and w if

$$2\begin{bmatrix} x & z \\ y & w \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 2 \end{bmatrix}.$$

(b) Find the determinant of the matrix

$$\begin{bmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{bmatrix}.$$

(c) Express the matrix

$$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

as the sum of a symmetric and a skew-symmetric matrix.

(d) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & \sin\theta & -\cos\theta \end{bmatrix}.$$

(e) Solve the following equations by matrix method (or by Cramer's rule) :

$$x + y + z = 3$$

$$x - 2y + 3z = 2$$

$$2x - y + z = 2$$

(f) Determine the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 0 & 3 & -2 \\ 2 & 4 & -3 \end{bmatrix}.$$

(g) Verify the given matrix

$$A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$$

satisfies its characteristic equation. Also find A^{-1}.

(h) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{bmatrix} 4 & 8 \\ 0 & -5 \end{bmatrix}.$$